Fernando, Pani W.; **Hausenblas, E.**; **Razafimandimby, Paul A.**

Irreducibility and exponential mixing of some stochastic hydrodynamical systems driven by pure jump noise. (English) Zbl 1354.60067

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Summary: In this paper, we derive several results related to the long-time behavior of a class of stochastic semilinear evolution equations in a separable Hilbert space **H**:

$$d\mathrm{u}(t) + [\mathrm{A}\mathrm{u}(t) + \mathbf{B}(\mathrm{u}(t), \mathrm{u}(t))]dt = dL(t), \quad \mathrm{u}(0) = x \in \mathrm{H}.$$

Here A is a positive self-adjoint operator and **B** is a bilinear map, and the driving noise L is basically a $D(\mathrm{A}^{-1/2})$-valued Lévy process satisfying several technical assumptions. By using a density transformation theorem type for Lévy measure, we first prove a support theorem and an irreducibility property of the Ornstein-Uhlenbeck processes associated to the nonlinear stochastic problem. Second, by exploiting the previous results we establish the irreducibility of the nonlinear problem provided that for a certain $\gamma \in [0, 1/4]$**B** is continuous on $D(\mathrm{A}^{\gamma}) \times D(\mathrm{A}^{\gamma})$ with values in $D(\mathrm{A}^{-1/2})$. Using a coupling argument, the exponential ergodicity is also proved under the stronger assumption that **B** is continuous on $\mathrm{H} \times \mathrm{H}$. While the latter condition is only satisfied by the nonlinearities of GOY and Sabra shell models, the assumption under which the irreducibility property holds is verified by several hydrodynamical systems such as the 2D Navier-Stokes, Magnetohydrodynamics equations, the 3D Leray-α model, the GOY and Sabra shell models.

MSC:

60H15	Stochastic partial differential equations (aspects of stochastic analysis)
60H10	Stochastic ordinary differential equations (aspects of stochastic analysis)
60H30	Applications of stochastic analysis (to PDEs, etc.)
60G51	Processes with independent increments; Lévy processes
60J75	Jump processes (MSC2010)

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stochastic semilinear evolution equations; separable Hilbert space; Lévy process; stochastic hydrodynamical systems; pure jump noise

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