Feng, X.

Regularity in Orlicz spaces for non-divergence degenerate elliptic equations on homogeneous groups. (English) Zbl 1352.35215

J. Contemp. Math. Anal., Armen. Acad. Sci. 51, No. 3, 111-120 (2016) and Izv. Nats. Akad. Nauk Armen., Mat. 51, No. 3, 41-55 (2016).

Summary: Let G be a homogeneous group, and let $X_1, X_2, \cdots, X_{p_0}$ be left-invariant real vector fields on G that are homogeneous of degree one with respect to the dilation group of G and satisfy Hörmander's condition. We establish a regularity result in the Orlicz spaces for the following equation:

$$Lu(x) = \sum_{j=1}^{p_0} a_{ij}(x) X_i X_J u(x) = f(x),$$

where $a_{ij}(x)$ are real valued, bounded measurable functions defined on G, satisfying the uniform ellipticity condition, and belonging to the space $VMO(G)$ with respect to the subelliptic metric induced by the vector fields $X_1, X_2, \cdots, X_{p_0}$.

MSC:

35R03 PDEs on Heisenberg groups, Lie groups, Carnot groups, etc.
35B65 Smoothness and regularity of solutions to PDEs
35J70 Degenerate elliptic equations

Keywords:

Orlicz estimate; homogeneous group; non-divergence degenerate elliptic equation

Full Text: DOI

References:

[1] R. A. Adams, J. J. F. Fournier, \textit{Sobolev Spaces} (Academic Press, New York, 2003). · Zbl 1098.46001

[2] Azroul, E.; Benkirane, A.; Tienari, M., On the regularity of solutions to the Poisson equations in Orlicz spaces, Bull. Belg.Math. Soc., 7, 1-12, (2000) · Zbl 0940.35061

[3] Bony, J. M., Principe du maximum, inégalitéde Harnack et uninitédu problème de Cauchy pour LES opérateurs elliptiques dégénérées, Ann. Inst. Fourier, 19, 227-304, (1969) · Zbl 0176.09703

[4] Bramanti, M.; Brandolini, L., Lp estimates for uniformly hypoelliptic operators with discontinuous coefficients on homogeneous groups, Rend. Sem. Mat. Univ. Pol. Torino, 58, 389-434, (2000) · Zbl 1072.35058

[5] Bramanti, M.; Cupini, G.; Lanconelli, E.; Priola, E., Global lp estimates for degenerate Ornstein-Uhlenbeck operators, Mathematische Zeitschrift, 266, 789-816, (2010) · Zbl 1206.35051

[6] Byun, S.; Yao, F.; Zhou, S., Gradient estimates in Orlicz space for nonlinear elliptic equations, J. Funct. Anal., 255, 1851-1873, (2008) · Zbl 1156.35038

[7] Cohn, W. S.; Lu, G.; Wang, P., Sub-elliptic global high order poincaréinequalities in stratified Lie groups and applications, J. Funct. Anal., 249, 393-424, (2007) · Zbl 1129.22007

[8] Folland, G. B., Subelliptic estimates and function spaces on nilpotent Lie groups, Arkiv forMath., 13, 161-207, (1975) · Zbl 0312.35026

[9] Gianazza, U., Regulariaty for nonlinear equations involving square Hömander operators, Nonlinear Anal., 23, 49-73, (1994) · Zbl 0868.47041

[10] Gogatishvili, A.; Kokilashvili, V., Criteria of weighted inequatities in Orlicz classes for maximal functions defined on homogeneous type spaces, GeorgianMathematical Journal, 1, 641-673, (1994) · Zbl 0819.42007

[11] J. Heinonen, \textit{Lectures on Analysis on Metric Spaces Universitext} (Springer, New York, 2001). · Zbl 0985.46008

[12] Hörmander, L., Hypoelliptic second order differential equations, Acta Math., 119, 147-171, (1967) · Zbl 0156.10701

[13] Jia, H.; Li, D.; Wang, L., Regularity theory in Orlicz spaces for the poissson equation, Manuscripta Math., 122, 265-275, (2007) · Zbl 1152.35019

[14] V. Kokilashvili, M. Krbec, \textit{Weighted Inequalities in Lorentz and Orlicz Spaces} (World Scientific, Singapore, 1991). ·

Zbl 0751.46021

[15] Krylov, N. V., Parabolic and elliptic equations with VMO coefficients, Commun. Partial Diff. Equs., 32, 453-475, (2007) · Zbl 1114.35079

[16] Krylov, N. V., Second-order elliptic equations with variably partially vmocoefficients, J.Funct.Anal., 257, 1695-1712, (2009) · Zbl 1171.35352

[17] Orlicz, W., Über eine gewisse klasse von Räumen vom typus B, Bull. Int. Acad. Pol., 8, 207-220, (1932) · Zbl 0006.31503

[18] M. Rao, Z. Ren, \textit{Applications of Orlicz Spaces} (Marcel Dekker, New York, 2000). · Zbl 0997.46027

[19] E. M. Stein, \textit{Harmonic Analysis: Real-Variable Methods, Orthogonality and Oscillatory Integrals} (Princeton Univ. Press, Princeton, New Jersey, 1993). · Zbl 0821.42001

[20] Wang, L., A geometric approach to the Calderón-Zygmund estimates, Acta Math. Sinica, 19, 383-396, (2003) · Zbl 1026.31003

[21] Wang, L.; Yao, F.; Zhou, S.; Jia, H., Optimal regularity for the Poisson equation, Proc. Amer.Math. Soc., 137, 2037-2047, (2009) · Zbl 1166.35011

[22] Weber, M., Stochastic processeswith values in exponential typeorlicz spaces, Ann. Prob., 16, 1365-1371, (1988) · Zbl 0656.60052