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Jacobi-Gauss-Lobatto collocation method for the numerical solution of $1+1$ nonlinear Schrödinger equations. (English) Zbl 1349.65511

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Summary: A Jacobi-Gauss-Lobatto collocation (J-GL-C) method, used in combination with the implicit Runge-Kutta method of fourth order, is proposed as a numerical algorithm for the approximation of solutions to nonlinear Schrödinger equations (NLSE) with initial-boundary data in $1+1$ dimensions. Our procedure is implemented in two successive steps. In the first one, the J-GL-C is employed for approximating the functional dependence on the spatial variable, using $(N-1)$ nodes of the Jacobi-Gauss-Lobatto interpolation which depends upon two general Jacobi parameters. The resulting equations together with the two-point boundary conditions induce a system of $2(N-1)$ first-order ordinary differential equations (ODEs) in time. In the second step, the implicit Runge-Kutta method of fourth order is applied to solve this temporal system. The proposed J-GL-C method, used in combination with the implicit Runge-Kutta method of fourth order, is employed to obtain highly accurate numerical approximations to four types of NLSE, including the attractive and repulsive NLSE and a Gross-Pitaevskii equation with space-periodic potential. The numerical results obtained by this algorithm have been compared with various exact solutions in order to demonstrate the accuracy and efficiency of the proposed method. Indeed, for relatively few nodes used, the absolute error in our numerical solutions is sufficiently small.

MSC:

65M70 Spectral, collocation and related methods for initial value and initial-boundary value problems involving PDEs

35Q55 NLS equations (nonlinear Schrödinger equations)

Keywords:

nonlinear complex Schrödinger equations; Gross-Pitaevskii equation; collocation method; Jacobi-Gauss-Lobatto quadrature; implicit Runge-Kutta method

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