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Characterizing width two for variants of treewidth. (English) [Zbl 1350.05116](#)
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Summary: In this paper, we consider the notion of special treewidth, recently introduced by *B. Courcelle* [*ibid.* 160, No. 6, 866–887 (2012; [Zbl 1236.05143](#))]. In a special tree decomposition, for each vertex v in a given graph, the bags containing v form a rooted path. We show that the class of graphs of special treewidth at most two is closed under taking minors, and give the complete list of the six minor obstructions. As an intermediate result, we prove that every connected graph of special treewidth at most two can be constructed by arranging blocks of special treewidth at most two in a specific tree-like fashion.

Inspired by the notion of special treewidth, we introduce three natural variants of treewidth, namely spaghetti treewidth, strongly chordal treewidth and directed spaghetti treewidth. All these parameters lie between pathwidth and treewidth, and we provide common structural properties on these parameters. For each parameter, we prove that the class of graphs having the parameter at most two is minor closed, and we characterize those classes in terms of a tree of cycles with additional conditions. Finally, we show that for each $k \geq 3$, the class of graphs with special treewidth, spaghetti treewidth, directed spaghetti treewidth, or strongly chordal treewidth, respectively at most k , is not closed under taking minors.

MSC:

- [05C69](#) Vertex subsets with special properties (dominating sets, independent sets, cliques, etc.) Cited in **3** Documents
- [05C70](#) Edge subsets with special properties (factorization, matching, partitioning, covering and packing, etc.)
- [05C05](#) Trees
- [03B15](#) Higher-order logic; type theory (MSC2010)

Keywords:

[treewidth](#); [special treewidth](#); [spaghetti treewidth](#); [strongly chordal treewidth](#); [minor](#)

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