Shutov, A. V.

Trigonometric integrals over one-dimensional quasilattices of arbitrary codimension. (English. Russian original) Zbl 1359.11071

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The author of this paper studies the one-dimensional quasilattices $Q = Q(\alpha, l_1, l_2)$ defined as the set of points $\{x_n\}_{n=-\infty}^{\infty}$ under the conditions $x_{-1} = 0$, $x_{n+1} = \{{x_n+l_1, n\alpha<1-\alpha, \atop x_n+l_2, n\alpha\geq 1-\alpha.}$, where l_1, l_2 are arbitrary distinct positive numbers, α is irrational number. This paper is a continuation and generalization of the previous paper [the author, Math. Notes 97, No. 5, 791–802 (2015; Zbl 1331.11062); translation from Mat. Zametki 97, No. 5, 781–793 (2015)]. The author studies trigonometric integrals of the form $I_{n,d} = \int_{\mathbb{T}^d}\int_0^1 |f_n(\alpha,\lambda)| d\lambda d\alpha$, where $\mathbb{T}^d$ is, as in the previous paper, exchanged tiling of the d-dimensional torus $\mathbb{T}^d = T_0 \bigsqcup T_1 \bigsqcup \cdots \bigsqcup T_d$. The main result is the following theorem: Let φ be a monotone increasing function such that $\varphi > 0$ for all positive integer n and the series $\Sigma_{n=1}^{\infty} 1/\varphi(n)$ converges. Then the following estimate holds: $I_{n,d} = O(\ln^{d+1} n\varphi(\ln\ln n))$. The paper ends with a posed problem about an inequality in the theory of Diophantine approximations.

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MSC:

11L03 Trigonometric and exponential sums, general
11H31 Lattice packing and covering (number-theoretic aspects)

Keywords:

quasilattice; trigonometric integrals; trigonometric sums; Diophantine approximation; exchanged tiling of d-dimensional torus

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