Thyssen, Pieter; Ceulemans, Arnout

Shattered symmetry. Group theory from the eightfold way to the periodic table. (English)

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There have been many books introducing group theoretic methods in physics and chemistry but this one is amazingly different. For one thing, even though it has serious mathematics, it is written like a story book. Throughout the text, footnotes are sprinkled, often with anecdotes. Furthermore, for lovers of Lewis Caroll's writings, this is a feast as there are plenty of references to them. The authors are experts in chemistry. Before going on to describe the contents etc., we quote from the first author Pieter Thyssen's Ph.D. thesis which gives a peek into the motivation of the authors in writing this book.

Thyssen says: "When I started PhD with my supervisor, Prof. Koen Binnemans, one thing was clear: we would study the contemporary problems related to the periodic system of the elements. How we would tackle these problems was still an open question however. For over one year, Koen and I went about collecting all the papers we could possibly find on the periodic table. We studied the quantum mechanics of the periodic system, looked into its information theoretical foundations, and read about the latest chemometric and nuclear approaches. But as the paper piles on my desk grew ever higher, we also became increasingly aware of the problems with these approaches. One small pile of papers on the group theory of the periodic system remained untouched however. They were written in a strange mathematical, and utterly incomprehensible language. One day in autumn, I decided to visit our library to hunt for an introductory book on group theory. Back in my office, I bravely started reading, but felt forced to stop on the very first page due to technical difficulties. Perhaps, I thought, I needed to start with something easier? Naively, I returned to the library the next day and borrowed two more books, but without avail. At the end of the week, I had assembled my own mini-library of cryptic books. Thumbing through these books, I started to believe group theory was reserved for a selected group of cognoscenti only, working in some impenetrable ivory tower. Baffled by the abstruse and subtle reasoning behind the wealth of formulae, and utterly bewildered and confused, I finally decided to visit my cosupervisor-to-be, Prof. Arnout Ceulemans. "Ah, Lie groups!", he exclaimed enthusiastically when peeking into the papers I presented him, "one of the topics that has always fascinated me." There and then, Arnout proposed to teach me the essentials of group theory...Thus started a most fruitful collaboration, which culminated in our book Shattered symmetry: from the eightfold way to the periodic table."

This book introduces Lie groups and Lie algebras, sketches their applications (which is the work of several prominent physicists over eight decades) to particle physics and finally, culminates in their own work – group theoretic explanation of the famed periodic system of Mendeleev. The book has three parts. The first one introduces group theory rather gently but with several interesting illustrations and allegories in the spirit of Lewis Caroll's books. The second part is on particle and quantum physics where Murray Gell-Mann's famous 'eightfold way' takes centre stage. In the third and final part, the authors try to develop similar analogies to apply to the famous problem of explaining the periodic table as a whole and, more specifically, to the elusive derivation/explanation of why the Madelung rule may be true.

Before going into specifics, we go into the basic motivation of looking at groups vis-a-vis the world of physics. Historically, Pierre Curie was perhaps the first to try to explain the relationship between physical properties and symmetry properties of a physical system. This was revealed through his studies on the thermal, electric and magnetic properties of crystals. He attempted to address the question as to which electric and magnetic phenomena will be allowed in a given crystalline medium possessing certain symmetry properties. In his 1894 work *Sur la symétrie dans les phénomènes physiques*, he argued that a phenomenon can exist in a medium that possesses its characteristic symmetry or that of one of its subgroups. The breaking of symmetry (colourfully alluded to in this book as "Shattering of symmetry") was again mentioned first by Pierre Curie. Very importantly, Pierre Curie stressed that asymmetry (that is, absence or reduction of symmetry) is why a phenomenon occurs. This means, in group-theoretic terms, that the initial symmetry group is "broken"/reduced to one of its subgroups. This makes it plausible that physical and chemical phenomena may be describable by studying relations between (representations of) transformation groups and (their restrictions to) some of their subgroups.

The book is meant mainly for chemists and physicists who wish to learn about applications of Lie groups but, mathematicians will also find it informative in many ways. It has some carefully researched history of group theory along with several interesting personal tidbits thrown in – for instance, the name of the person who may have been the cause of Galois's infamous duel – with beautiful illustrations. More than that, for mathematicians like me who are not very well informed vis-a-vis applications of group representations to physics and chemistry, this is an excellent opportunity to grasp the essentials.

The first 135 pages introduce group theory and discuss the Lie groups O(2), SO(2), SO(3), SU(2) and their Lie algebras. The Cartan-Weyl theory is also described succinctly. The second part consisting of the next 135 pages is about the particle zoo (as the authors call it). The fascinating history of discovery of elementary particles and their gradual understanding in terms of the ubiquitous (in physics) Lie group SU(3) is penned in a lively manner. The phrase"eightfold way" of the title is due to Murray Gell-Mann and the authors go through the fascinating developments which led to this work and beyond.

Finally, the authors come to their pet project which is a group theoretic explanation/understanding of Mendeleev's periodic table. As is well-documented, Mendeleev was bold enough to predict the existence of several new elements just because of the scheme he had devised to classify the known ones. In elementary particle physics, the compact Lie group SU(3) is the "omniscient" group – or more precisely, the decompositions of restriction of tensor products of representations of the the Lie algebra su(3) to subalgebras is the carrier of all sorts of information. To cut a long story short, analogous to the classification in particle physics, the authors put forth a candidate which should play a corresponding role in chemistry, vis-a-vis the periodic table. They argue that the symmetry breaking involves the chain of groups

$$\mathrm{SO}(4,2)\times\mathrm{SU}(2)\supset\mathrm{SO}(3,2)\times\mathrm{SU}(2)\supset\mathrm{SO}'(4)\times\mathrm{SU}(2).$$

Note that $\mathrm{SO}(4,2)$ is non-compact.

While the authors give a number of details to convince that this is a good candidate to explain the symmetry breaking in their situation, they also point out to critiques by other chemists on certain other aspects which need to be explained. Two crucial aspects of the periodic table are the Madelung $(n+l,n)$-rule that is believed to rationalize the orbital filling order in many electron systems and the 'doubling of the periods' which is a consequence of the Madelung rule. In other words, the Madelung $(n+l,n)$-rule predicts that with increasing nuclear charge, one-electron orbitals are filled according to increasing $n+l$ (here, n is the principal quantum number and l is the orbital quantum number). For fixed $n+l$, the orbitals are filled according to increasing n. The Madelung rule itself has been under attack by various chemists like E. W. H. Schwarz who remarked that "this approximate rule of thumb has been at variance with too many facts" and referred to it as "the $n+l$ blunder". The authors of this book state: "Let it be clear at the outset that we do not agree with Schwarz's conclusions". While the authors admit that Schwarz has raised a number of pertinent issues which shows the need for a correct interpretation of the Madelung rule, they say that its explanatory power cannot be underestimated. In Chapter 13, the authors mention that there are two types of group theoretic approaches – the atomic physics approach and the elementary particle approach. Where one has periodic systems where the Hamiltonian is difficult to construct, a phenomenological approach is advocated wherein symmetry groups are not derived from first principles but postulated from empirical data.

The authors show in Chapter 13 that if one identifies the overall symmetry group of the bound states of the hydrogen atom with the direct product of $\mathrm{SO}(4,2)$ and $\mathrm{SU}(2)$, then the collection of all possible (n,l) combinations, representing different chemical elements, forms a basis for an infinite-dimensional irreducible unitary representation $h\otimes[2]$ of $\mathrm{SO}(4,2)\times\mathrm{SU}(2)$ (where $[2]$ is the fundamental representation of $\mathrm{SU}(2)$). They also show that on restricting to the anti-de Sitter group $\mathrm{SO}(3,2)$, the infinite-dimensional manifold of chemical elements splits into two sets: one with $n+l$ odd and the other with $n+l$ even; this leads to the doubling of the Aufbau series. However, as the authors admit at the end of this chapter, the approach did not succeed in finding a group theoretic structure for the Madelung $(n+l,n)$-rule.

The authors refer to the group $\mathrm{SO}(4,2)$ as the "group which provides a window to the whole of chemistry." As mentioned above, they identify the overall symmetry group of the periodic table with the product of this group with $\mathrm{SU}(2)$ where $\mathrm{SU}(2)$ accounts for the spin part. Thus, the authors provide a global group structure to the periodic system and interpret the period doubling phenomenon. Finally, by introducing a new operator which yields a new $\mathrm{so}'(4,2)$-algebra, they are able to construct Madelung operators which form a nonlinear Lie algebra. The authors conclude that giving up the requirement of linearity should not be deplored as a loss but welcomed as a new opportunity!

Finally, mathematicians would realize right away that this book is not written by mathematicians – not because of any lack of expertise in mathematics – because the authors' names do not appear in alphabetical order!

Reviewer: Balasubramanian Sury (Bangalore)

MSC:

20-01	Introductory exposition (textbooks, tutorial papers, etc.) pertaining to group theory	Cited in **1** Document
22-01	Introductory exposition (textbooks, tutorial papers, etc.) pertaining to topological groups	
20G20	Linear algebraic groups over the reals, the complexes, the quaternions	
20F65	Geometric group theory	
92E10	Molecular structure (graph-theoretic methods, methods of differential topology, etc.)	
22E70	Applications of Lie groups to the sciences; explicit representations	
17B81	Applications of Lie (super)algebras to physics, etc.	

Keywords:

Mendeleev's periodic table; Madelung rule; elementary particle physics; Lie groups; Lie algebras; SU(3); $SO(4,2)$; tensor product; Lewis Caroll; period doubling