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**On finiteness of curves with high canonical degree on a surface.** (English) Zbl 1360.14019
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By exploiting a result of *Y. Miyaoka* [Publ. Res. Inst. Math. Sci. 44, No. 2, 403–417 (2008; Zbl 1162.14026)] this paper obtains a simple explicit bound on the canonical degree $k_C = K_X.C$ of a curve $C \neq \mathbb{P}^1$ of negative self-intersection on a smooth complex projective surface $X$ of non-negative Kodaira dimension, in terms of the geometric genus $g$ of $C$ and the invariants of $X$. In fact the only invariant needed is $a = 3c_2 - K_X^2$. Then

$$k_C \leq 3(g-1) + \frac{3}{4}a + \frac{1}{4}\sqrt{9a^2 + 24a(g-1)}.$$

Moreover, for $X$ of general type, there are finitely many curves with $k_C \geq 3 + \epsilon(g-1) \geq 0$. This (together with some refinements omitted here for brevity) yields several corollaries. One is that a Shimura surface contains only finitely many Shimura curves: this is known, but the proof here is more economical both mathematically and in workforce terms than the one in [*T. Bauer* et al., Duke Math. J. 162, No. 10, 1877–1894 (2013; Zbl 1272.14009)]. That paper is largely concerned with questions of bounded negativity (crudely, $C^2$ should be bounded below) and leads the present authors to rephrase their results so as to obtain partial results of that type. In particular they address a conjecture of Nagata that there are no negative curves apart from $-1$-curves on $\mathbb{P}^2$ blown up in $n \geq 10$ general points, using an extension of their result to some cases of negative Kodaira dimension, and a conjecture of Vojta that $k_C \leq (4+\epsilon)(g-1) + B(\epsilon)$. In both cases they get interesting results, informative but weaker than what is conjectured, using relatively direct methods.

Reviewer: G. K. Sankaran (Bath)

### MSC:

14C17 Intersection theory, characteristic classes, intersection multiplicities in algebraic geometry
14G35 Modular and Shimura varieties
14J29 Surfaces of general type

### Keywords:

bounded negativity conjecture; Nagata's conjecture; Vojta's conjecture

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