Many applications of fuzzy sets restrict attention to the convenient metric space $(\mathcal{E}^n, D)$ of normal, fuzzy convex sets on the base space $\mathbb{R}^n$, with D the supremum over the Hausdorff distances between corresponding level sets. We mention in particular the fuzzy random variables of *M. L. Puri* and *D. A. Ralescu* [Ann. Probab. 13, 1373-1379 (1985; Zbl 0583.60011)], the fuzzy differential equations of *O. Kaleva* [Fuzzy Sets Syst. 24, 301-317 (1987; Zbl 0646.34019)], the fuzzy dynamical systems of the second author [Fuzzy Sets Syst. 7, 275-296 (1982; Zbl 0509.54040)] and the chaotic iterations of fuzzy sets of Diamond and Kloeden. In these papers specific results are often obtained for compact subsets of $\mathcal{E}^n$, which raises the question of how to characterize such compact subsets. The purpose of this is to present a convenient characterization of compact subsets of the metric space $(\mathcal{E}^n, D)$. Our main result is that a closed subset of $\mathcal{E}^n$ is compact if and only if the support sets are uniformly bounded in $\mathbb{R}^n$ and the support functions of Puri and Ralescu are equileftcontinuous in the membership grade variable α uniformly on the unit sphere S^{n-1} of $\mathbb{R}^n$. To this end we note that the support functions provides a means of embedding all of the space $\mathcal{E}^n$ in a Banach space, which we exhibit explicitly, not just the subspace $\mathcal{E}^n_{Lip}$ of 'Lipschitzian' fuzzy sets considered by Puri and Ralescu.

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References:

[1] P. Diamond, Fuzzy chaos, \textit{J. Math. Anal. Appl.} (submitted). · Zbl 0921.93026

[2] Graves, L.M, The theory of functions of real variables, (1946), McGraw-Hill New York · Zbl 0063.01720

[3] Kaleva, O, On the convergence of fuzzy sets, Fuzzy sets and systems, 17, 54-65, (1985) · Zbl 0584.54004

[4] O. Kaleva, The Cauchy problem for fuzzy differential equations, Preprint. · Zbl 0696.34005

[5] Kloeden, P.E, Fuzzy dynamical systems, Fuzzy sets and systems, 7, 275-296, (1982) · Zbl 0509.54040

[6] Kloeden, P.E, Chaotic mappings on fuzzy sets, (), Preprint · Zbl 0746.54010

[7] Kolmogorov, A.N; Fomin, S.V, Introductory real analysis, (1975), Dover New York · Zbl 0213.07305

[8] Puri, M.L; Ralescu, D.A, The concept of normality for fuzzy random variables, Ann. probub., 13, 1373-1379, (1985) · Zbl 0583.60011