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Ultrarelativistic bound states in the spherical well. (English) Zbl 1342.81130

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Summary: We address an eigenvalue problem for the ultrarelativistic (Cauchy) operator $(-\Delta)^{1/2}$, whose action is restricted to functions that vanish beyond the interior of a unit sphere in three spatial dimensions. We provide high accuracy spectral data for lowest eigenvalues and eigenfunctions of this infinite spherical well problem. Our focus is on radial and orbital shapes of eigenfunctions. The spectrum consists of an ordered set of strictly positive eigenvalues which naturally splits into non-overlapping, orbitally labelled $E_{(k,l)}$ series. For each orbital label $l = 0, 1, 2, \ldots$, the label $k = 1, 2, \ldots$ enumerates consecutive lth series eigenvalues. Each of them is $2l + 1$-degenerate. The $l = 0$ eigenvalues series $E_{(k,0)}$ are identical with the set of even labeled eigenvalues for the $d = 1$ Cauchy well: $E_{(k,0)}(d = 3) = E_{2k}(d = 1)$. Likewise, the eigenfunctions $\psi_{(k,0)}(d = 3)$ and $\psi_{2k}(d = 1)$ show affinity. We have identified the generic functional form of eigenfunctions of the spherical well which appear to be composed of a product of a solid harmonic and of a suitable purely radial function. The method to evaluate (approximately) the latter has been found to follow the universal pattern which effectively allows to skip all, sometimes involved, intermediate calculations (those were in usage, while computing the eigenvalues for $l \leq 3$).

MSC:

81Q10 Selfadjoint operator theory in quantum theory, including spectral analysis
35P05 General topics in linear spectral theory for PDEs

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