Pham, Kim

An energetic formulation of a one-dimensional model of superelastic SMA. (English)

Zbl 1341.74027
Contin. Mech. Thermodyn. 26, No. 6, 833-857 (2014).

Summary: This paper presents an energetic framework for the study of the macroscopic evolution of shape memory alloys (SMA) with softening behavior. It is written for a class of standard rate-independent materials with an internal variable derived from the Drucker-Ilyushin work property. This one-dimensional model is defined by three material functions of the internal variable and one material parameter. The quasi-static evolution is formulated for a one-dimensional bar under traction and is based on two physical principles: a stability criterion which consists in selecting the local minima of the total energy and an energy balance condition which requires the absolute continuity of the total energy. The stability criterion aims to overcome the non-uniqueness issue associated with the intrinsic softening character of SMA while the energy balance condition accounts for evolutions even with possible time discontinuities. While being consistent with the classical Kuhn-Tucker formulation of the phase transformations, such energetic formulation proved to be more suitable than this latter for the study of stress-softening SMA. Both homogeneous and non-homogenous solutions are investigated with respect to this variational evolution problem. Specifically, we show the instability of the homogeneous states for softening materials and construct, in this latter case, a non-homogeneous stable evolution that follows a transformation stress line which corresponds to the Maxwell line of the softening intrinsic behavior.

MSC:

74B20 Nonlinear elasticity
74N10 Displacive transformations in solids
74C10 Small-strain, rate-dependent theories of plasticity (including theories of viscoplasticity)

Keywords:

shape memory alloys (SMA); superelasticity; softening; one-dimensional model; energetic approach; stability criterion

Full Text: DOI

References:

[1] Abeyaratne, R.; Knowles, J.K., A continuum model of a thermoelastic solid capable of undergoing phase transitions, J. Mech. Phys. Solids, 41, 541-571, (1993) · Zbl 0825.73058

[2] Auricchio, F.; Mielke, A.; Stefanelli, U., A rate-independent model for the isothermal quasi-static evolution of shape-memory materials, Math. Models Methods Appl. Sci., 18, 125-164, (2008) · Zbl 1151.74319

[3] Auricchio, F.; Sacco, E., A one-dimensional model for superelastic shape-memory alloys with different elastic properties between austenite and martensite, Int. J. Non-Linear Mech., 32, 1101-1114, (1997) · Zbl 0906.73006

[4] Bourdin, B.; Francfort, G.A.; Marigo, J.-J., The variational approach to fracture, J. Elast., 91, 5-148, (2008) · Zbl 1176.74018

[5] Boyd, J.; Lagoudas, D., A thermodynamical constitutive model for shape memory materials. part i. the monolithic shape memory alloy, Int. J. Plast., 12, 805-842, (1996) · Zbl 0898.73020

[6] Coleman, B.D.; Noll, W., The thermodynamics of elastic materials with heat conduction and viscosity, Arch. Ratl. Mech. Anal., 13, 167-178, (1963) · Zbl 0113.17802

[7] Dal Maso, G.; DeSimone, A.; Mora, M.G., Quasistatic evolution problems for linearly elastic-perfectly plastic materials, Arch. Ratl. Mech. Anal., 180, 237-291, (2006) · Zbl 1093.74007

[8] Duval, A.; Haboussi, M.; Ben Zineb, T., Modelling of localization and propagation of phase transformation in superelastic SMA by a gradient nonlocal approach, Int. J. Solids Struct., 48, 1879-1893, (2011)

[9] Ericksen, J., Equilibrium of bars, J. Elast., 5, 191-201, (1975) · Zbl 0324.73067

[10] Feng, P.; Sun, Q.-P., Experimental investigation on macroscopic domain formation and evolution in polycrystalline niti microtubing under mechanical force, J. Mech. Phys. Solids, 54, 1568-1603, (2006)

[11] Hallai, J.F.; Kyriakides, S., Underlying material response for lüders like instabilities, Int. J. Plast., 47, 1-12, (2013)

[12] Halphen, B.; Nguyen, Q.S., Sur LES matériaux standard généralisés, J. Méca., 14, 39-63, (1975) · Zbl 0308.73017

[13] He, Y.; Sun, Q., Macroscopic equilibrium domain structure and geometric compatibility in elastic phase transition of thin plates, Int. J. Mech. Sci., 52, 198-211, (2010)

[14] Huo, Y.; Müller, I., Nonequilibrium thermodynamics of pseudoelasticity, Contin. Mech. Thermodyn., 5, 163-204, (1993) · Zbl 0780.73006

[15] Idesman, A.; Levitas, V.; Preston, D.; Cho, J.-Y., Finite element simulations of martensitic phase transitions and microstructures based on a strain softening model, J. Mech. Phys. Solids, 53, 495-523, (2005) · Zbl 1122.74475

[16] Ivshin, Y.; Pence, T.J., A thermomechanical model for a one variant shape memory material, J. Intell. Mater. Syst. Struct., 5, 455-473, (1994)

[17] Lagoudas, D.; Hartl, D.; Chemisky, Y.; Machado, L.; Popov, P., Constitutive model for the numerical analysis of phase transformation in polycrystalline shape memory alloys, Int. J. Plast., 32, 155-183, (2012)

[18] Liang, C.; Rogers, C., One-dimensional thermomechanical constitutive relations for shape memory materials, J. Intell. Mater. Syst. Struct., 1, 207-234, (2012)

[19] Marigo, J.-J., Constitutive relations in plasticity, damage and fracture mechanics based on a work property, Nucl. Eng. Des., 114, 249-272, (1989)

[20] Mazars, J.; Berthaud, Y., Une technique expérimentale appliquée au béton pour créer un endommagement diffus et mettre en évidence son caractère unilatéral, C.R. Acad. Sci., 308, 579-584, (1989)

[21] Mielke, A., Evolution of rate-independent systems, Handb. Differ. Equ. Evol. Equ., 2, 461-559, (2006) · Zbl 1120.47062

[22] Morin, C.; Moumni, Z.; Zaki, W., Thermomechanical coupling in shape memory alloys under cyclic loadings: experimental analysis and constitutive modeling, Int. J. Plast., 27, 1959-1980, (2011) · Zbl 1442.74046

[23] Patoor, E., Eberhardt, A., Berveiller, M.: Thermomechanical behavior of shape memory alloys. In: European Symposium on Martensitic Transformations, pp. 133-140. EDP Sciences (1989) · Zbl 0825.73058

[24] Pham, K.; Marigo, J.-J., The variational approach to damage: I. the foundations, C.R. Meca. Acad. Sci., 338, 191-198, (2010) · Zbl 1300.74046

[25] Pham, K.; Marigo, J.-J.; Maurini, C., The issues of the uniqueness and the stability of the homogeneous response in uniaxial tests with gradient damage models, J. Mech. Phys. Solids, 59, 1163-1190, (2011) · Zbl 1270.74015

[26] Popov, P.; Lagoudas, D.C., A 3-d constitutive model for shape memory alloys incorporating pseudoelasticity and detwinning of self-accommodated martensite, Int. J. Plast., 23, 1679-1720, (2007) · Zbl 1127.74008

[27] Rajagopal, K.; Srinivasa, A., On the thermomechanics of shape memory wires, Z. Angew. Math. Phys., 50, 459-496, (1999) · Zbl 0951.74005

[28] San Juan, J.; Nó, M.; Schuh, C., Nanoscale superelastic alloys with ultra-high damping capacity, Nat. Nanotechnol., 4, 415-419, (2009)

[29] Shaw, J.A.; Kyriakides, S., Thermomechanical aspects of niti, J. Mech. Phys. Solids, 43, 1243-1281, (1995)

[30] Shaw, J.A.; Kyriakides, S., On the nucleation and propagation of phase transformation fronts in a niti alloy, Acta Mater., 45, 683-700, (1997)

[31] Shioya, T.; Shioiri, J., Elastic-plastic analysis of the yield process in mild steel, J. Mech. Phys. Solids, 24, 187-204, (1976)

[32] Song, Z., Dai, H.H., Sun, Q.P.: Propagation stresses in phase transitions of an SMA wire: new analytical formulas based on an internal-variable model. Int. J. Plast. \textbf{42}, 101-119 (2013)

[33] Sun, Q., He, Y.: A multiscale continuum model of the grain-size dependence of the stress hysteresis in shape memory alloy polycrystals. Int. J. Solids Struct. \textbf{45}(13), 3868-3896, 2008. Special Issue Honoring K.C. Hwang Fracture, Plasticity, Micro- and Nanomechanics Special Issue Honoring K.C. Hwang · Zbl 1169.74519

[34] Sun, Q.P.; Hwang, K.C., Micromechanics modelling for the constitutive behavior of polycrystalline shape memory alloysñi. derivation of general relations, J. Mech. Phys. Solids, 41, 1-17, (1993) · Zbl 0767.73062

[35] Sun, Q.P.; Hwang, K.C., Micromechanics modelling for the constitutive behavior of polycrystalline shape memory alloysñii. study of the individual phenomena, J. Mech. Phys. Solids, 41, 19-33, (1993) · Zbl 0767.73063

[36] Sun, Q.P.; Zhao, H.; Zhou, R.; Saletti, D.; Yin, H., Recent advances in spatiotemporal evolution of thermomechanical fields during the solid-solid phase transition, C.R. Meca., 340, 349-358, (2012)

[37] Thamburaja, P.; Anand, L., Polycrystalline shape-memory materials: effect of crystallographic texture, J. Mech. Phys. Solids, 49, 709-737, (2001) · Zbl 1011.74049

[38] Thamburaja, P.; Anand, L., Superelastic behavior in tension-torsion of an initially-textured ti-ni shape-memory alloy, Int. J. Plast., 18, 1607-1617, (2002) · Zbl 1032.74610

[39] Tobushi, H., Tanaka, K., Hori, T., Sawada, T., Hattori, T.: Pseudoelasticity of tini shape memory alloy: dependence on maximum strain and temperature. JSME Int. J. Ser. A, Mech. Mater. Eng. \textbf{36}(3), 314-318, Jul 1993. ISSN 09148809 · Zbl 0113.17802

[40] Zaki, W.; Moumni, Z., A three-dimensional model of the thermomechanical behavior of shape memory alloys, J. Mech. Phys. Solids, 55, 2455-2490, (2007) · Zbl 1171.74032

[41] Ziolkowski, A., Three-dimensional phenomenological thermodynamic model of pseudoelasticity of shape memory alloys at finite strains, Contin. Mech. Thermodyn., 19, 379-398, (2007) · Zbl 1160.74322

paper as accurately as possible without claiming the completeness or perfect precision of the matching.