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**Dynamical insurance models with investment: constrained singular problems for integrodifferential equations.** (English. Russian original) Zbl 1349.91129  
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Summary: Previous and new results are used to compare two mathematical insurance models with identical insurance company strategies in a financial market, namely, when the entire current surplus or its constant fraction is invested in risky assets (stocks), while the rest of the surplus is invested in a risk-free asset (bank account). Model I is the classical Cramér-Lundberg risk model with an exponential claim size distribution. Model II is a modification of the classical risk model (risk process with stochastic premiums) with exponential distributions of claim and premium sizes. For the survival probability of an insurance company over infinite time (as a function of its initial surplus), there arise singular problems for second-order linear integrodifferential equations (IDEs) defined on a semiinfinite interval and having nonintegrable singularities at zero: model I leads to a singular constrained initial value problem for an IDE with a Volterra integral operator, while II model leads to a more complicated nonlocal constrained problem for an IDE with a non-Volterra integral operator. A brief overview of previous results for these two problems depending on several positive parameters is given, and new results are presented. Additional results are concerned with the formulation, analysis, and numerical study of “degenerate” problems for both models, i.e., problems in which some of the IDE parameters vanish; moreover, passages to the limit with respect to the parameters through which we proceed from the original problems to the degenerate ones are singular for small and/or large argument values. Such problems are of mathematical and practical interest in themselves. Along with insurance models without investment, they describe the case of surplus completely invested in risk-free assets, as well as some noninsurance models of surplus dynamics, for example, charity-type models.

**MSC:**

- 91B30 Risk theory, insurance (MSC2010)
- 45J05 Integro-ordinary differential equations
- 34B16 Singular nonlinear boundary value problems for ordinary differential equations
- 60H30 Applications of stochastic analysis (to PDEs, etc.)

Cited in **3** Documents

**Keywords:**

Cramér-Lundberg-type dynamical insurance models with deterministic and stochastic premiums; exponential distributions of premium and claim sizes; investments in risky and risk-free assets; survival probability of an insurance company as a function of its initial surplus; second-order linear IDEs on a half-line with Volterra and non-Volterra integral operators; singular initial value and nonlocal constrained problems; degenerate problems; related singular problems for ordinary differential equations; existence; uniqueness; behavior of solutions; numerical solution algorithms; numerical results; comparison of models

**Full Text:** [DOI](#)

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