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Spaces of algebraic maps from real projective spaces to toric varieties. (English)

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Given manifolds X and Y, write $\mathrm{Map}^*(X,Y)$ for the space of bas-point preserving continuous maps from X to Y. If X and Y are complex manifolds, *J. Mostovoy* [Topology 45, No. 2, 281–293 (2006; Zbl 1086.58005); Q. J. Math. 63, No. 1, 181–187 (2012; Zbl 1237.58012)] determined an integer n_D such that the inclusion map $j_D : \mathrm{Hol}^*_D(X,Y) \to \mathrm{Map}^*(X,Y)$ is a homology equivalence through dimension n_D, where $D = (d_1,\dots,d_r)$ is a tuple of integers and $\mathrm{Hol}^*_D(X,Y)$ denotes the space of holomorphic maps from X to Y of degree D. Recently, *J. Mostovoy* and *E. Munguia-Villanueva* [Spaces of morphisms from a projective space to a toric variety, preprint, arXiv:1210.2795] generalized that result to the case of holomorphic maps from a complex projective space $\mathbb{C}P^m$ to a compact smooth toric variety X_Σ associated to a fan Σ.

Given algebraic varieties X and Y, write $\mathrm{Alg}^*_D(X,Y)$ for the space of algebraic (regular) maps from X to Y of degree D and $A_D(X,Y)$ for the space of tuples of polynomials representing elements of $\mathrm{Alg}^*_D(X,Y)$. For a real projective space $\mathbb{R}P^m$, the authors study the natural map $i_D : A_D(m,X_\Sigma) \to \mathrm{Map}^*(X,Y)$ and show that the induced map $i'_D : A_D(m,X_\Sigma;g) \to F(\mathbb{R}R^m,X_\Sigma;g) \simeq \Omega^m X_\Sigma$ is a homology equivalence through a dimension $n_D(d_1,\dots,d_r;m)$.

Reviewer: Marek Golasiński (Olsztyn)

MSC:

55R80	Discriminantal varieties and configuration spaces in algebraic topology	Cited in **3** Documents
14M25	Toric varieties, Newton polyhedra, Okounkov bodies	
55P10	Homotopy equivalences in algebraic topology	
55P35	Loop spaces	

Keywords:

algebraic map; fan; homogenous coordinate; primitive element; rational polyhedral cone; simplicial resolution; toric variety; Vassiliev spectral sequence

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