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Analysis of stability, verification and chaos with the Kreiss-Yström equations. (English)

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Summary: A system of two coupled PDEs originally proposed and studied by *H. O Kreiss* and *J. Yström* [Math. Comput. Modelling 35, No. 11–12, 1271–1295 (2002; Zbl 1066.76064)], which is dynamically similar to a one-dimensional two-fluid model of two-phase flow, is investigated here. It is demonstrated that in the limit of vanishing viscosity (i.e., neglecting second-order and higher derivatives), the system possesses complex eigenvalues and is therefore ill-posed. The regularized problem (i.e., including viscous second-order derivatives) retains the long-wavelength linear instability but with a cut-off wavelength, below which the system is linearly stable and dissipative. A second-order accurate numerical scheme, which is verified using the method of manufactured solutions, is used to simulate the system. For short to intermediate periods of time, numerical solutions compare favorably to those published previously by the original authors. However, the solutions at a later time are considerably different and have the properties of chaos. To quantify the chaos, the largest Lyapunov exponent is calculated and found to be approximately 0.38. Additionally, the correlation dimension of the attractor is assessed, resulting in a fractal dimension of 2.8 with an embedded dimension of approximately 6. Subsequently, the route to chaos is qualitatively explored with investigations of asymptotic stability, traveling-wave limit cycles and intermittency. Finally, the numerical solution, which is grid-dependent in space-time for long times, is demonstrated to be convergent using the time-averaged amplitude spectra.

MSC:

35Q35	PDEs in connection with fluid mechanics
35B25	Singular perturbations in context of PDEs
35K40	Second-order parabolic systems
35R25	Ill-posed problems for PDEs
37N15	Dynamical systems in solid mechanics
76Txx	Multiphase and multicomponent flows

Keywords:

ill-posed; verification; chaos; two-fluid model

Full Text: DOI

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