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On a Gerber-Shiu type function and its applications in a dual semi-Markovian risk model.
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Summary: In this paper, we consider a dual risk process which can be used to model the surplus of a business that invests money constantly and earns gains randomly in both time and amount. The occurrences of the gains and their amounts are assumed follow a semi-Markovian structure. We analyze a quantity resembling the Gerber-Shiu expected discounted penalty function [*H. U. Gerber and E. S. W. Shiu*, *N. Am. Actuar. J.* 2, No. 1, 48–78 (1998; [Zbl 1081.60550](#))] that incorporates random variables defined before and after the time of ruin, such as the minimum surplus level before ruin and the time of the first gain after ruin. General properties of the function are studied, and some exact results are derived upon exponential distributional assumptions on either the inter-arrival times or the gain amounts. Applications in a perpetual insurance and the last inter-arrival time containing the time of ruin are given along with some numerical examples.

MSC:

[60K15](#) Markov renewal processes, semi-Markov processes
[91B30](#) Risk theory, insurance (MSC2010)

Cited in **2** Documents

Keywords:

dual risk model; semi-Markovian risk process; Gerber-Shiu function; generalized penalty function; perpetual insurance; last inter-arrival time

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References:

- [1] Adan, I. J.B. F.; Kulkarni, V. G., Single-server queue with Markov-dependent inter-arrival and service times, *Queueing Syst.*, 45, 2, 113-134, (2003) · [Zbl 1036.90029](#)
- [2] Ahn, S.; Badescu, A. L., On the analysis of the gerber-shiu discounted penalty function for risk processes with Markovian arrivals, *Insurance Math. Econom.*, 41, 2, 234-249, (2007) · [Zbl 1193.60103](#)
- [3] Albrecher, H.; Badescu, A. L.; Landriault, D., On the dual risk model with tax payments, *Insurance Math. Econom.*, 42, 3, 1086-1094, (2008) · [Zbl 1141.91481](#)
- [4] Albrecher, H.; Boxma, O. J., On the discounted penalty function in a Markov-dependent risk model, *Insurance Math. Econom.*, 37, 3, 650-672, (2005) · [Zbl 1129.91023](#)
- [5] Albrecher, H.; Gerber, H. U.; Yang, H., A direct approach to the discounted penalty function, *N. Am. Actuar. J.*, 14, 4, 420-434, (2010) · [Zbl 1219.91063](#)
- [6] Albrecher, H.; Teugels, J. L., Exponential behavior in the presence of dependence in risk theory, *J. Appl. Probab.*, 43, 1, 257-273, (2006) · [Zbl 1097.62110](#)
- [7] Asmussen, S., *Applied probability and queues*, (2003), Springer New York · [Zbl 1029.60001](#)
- [8] Avanzi, B.; Cheung, E. C.K.; Wong, B.; Woo, J.-K., On a periodic dividend barrier strategy in the dual model with continuous monitoring of solvency, *Insurance Math. Econom.*, 52, 1, 98-113, (2013) · [Zbl 1291.91088](#)
- [9] Avanzi, B.; Gerber, H. U.; Shiu, E. S.W., Optimal dividends in the dual model, *Insurance Math. Econom.*, 41, 1, 111-123, (2007) · [Zbl 1131.91026](#)
- [10] Badescu, A. L.; Breuer, L.; Da Silva Soares, A.; Latouche, G.; Remiche, M.-A.; Stanford, Risk processes analyzed as fluid queues, *Scand. Actuar. J.*, 2005, 2, 127-141, (2005) · [Zbl 1092.91037](#)
- [11] Biffis, E.; Morales, M., On a generalization of the gerber-shiu function to path-dependent penalties, *Insurance Math. Econom.*, 46, 1, 92-97, (2010) · [Zbl 1231.91146](#)
- [12] Boikov, A. V., The cramer-lundberg model with stochastic premium process, *Theory Probab. Appl.*, 47, 3, 489-493, (2002) · [Zbl 1033.60093](#)
- [13] Cai, J.; Feng, R.; Willmot, G. E., On the total discounted operating costs up to default and its applications, *Adv. Appl. Probab.*, 41, 2, 495-522, (2009) · [Zbl 1173.91023](#)
- [14] Cheung, E. C.K., Discussion of 'recursive calculation of the dividend moments in a multi-threshold risk model', *N. Am.*

- Actuar. J., 12, 3, 336-340, (2008)
- [15] Cheung, E. C.K., A unifying approach to the analysis of business with random gains, *Scand. Actuar. J.*, 2012, 3, 153-182, (2012) · [Zbl 1277.60148](#)
- [16] Cheung, E. C.K.; Drekić, S., Dividend moments in the dual risk model: exact and approximate approaches, *ASTIN Bull.*, 38, 2, 399-422, (2008) · [Zbl 1256.91026](#)
- [17] Cheung, E. C.K.; Feng, R., A unified analysis of claim costs up to ruin in a Markovian arrival risk model, *Insurance Math. Econom.*, 53, 1, 98-109, (2013) · [Zbl 1284.91214](#)
- [18] Cheung, E. C.K.; Landriault, D., Analysis of a generalized penalty function in a semi-Markovian risk model, *N. Am. Actuar. J.*, 13, 4, 497-513, (2009)
- [19] Cheung, E. C.K.; Landriault, D., A generalized penalty function with the maximum surplus prior to ruin in a MAP risk model, *Insurance Math. Econom.*, 46, 1, 127-134, (2010) · [Zbl 1231.91156](#)
- [20] Cheung, E. C.K.; Landriault, D.; Badescu, A. L., On a generalization of the risk model with Markovian claim arrivals, *Stoch. Models*, 27, 3, 407-430, (2011) · [Zbl 1237.91124](#)
- [21] Cheung, E. C.K.; Landriault, D.; Willmot, G. E.; Woo, J.-K., Gerber-shiu analysis with a generalized penalty function, *Scand. Actuar. J.*, 2010, 3, 185-199, (2010) · [Zbl 1226.60123](#)
- [22] Cheung, E. C.K.; Landriault, D.; Willmot, G. E.; Woo, J.-K., Structural properties of gerber-shiu functions in dependent sparre Andersen models, *Insurance Math. Econom.*, 46, 1, 117-126, (2010) · [Zbl 1231.91157](#)
- [23] Cheung, E. C.K.; Landriault, D.; Willmot, G. E.; Woo, J.-K., On orderings and bounds in a generalized sparre Andersen risk model, *Appl. Stoch. Models Bus. Ind.*, 27, 1, 51-60, (2011) · [Zbl 1274.60050](#)
- [24] Çinlar, E., Markov renewal theory, *Adv. Appl. Probab.*, 1, 2, 123-187, (1969) · [Zbl 0212.49601](#)
- [25] H. Cramér, *Collective Risk Theory*. Jubilee volume of Forsakringsbolaget Skandia, Stockholm, 1955.
- [26] de Smit, J. H.A., Explicit Wiener-Hopf factorizations for the analysis of multidimensional queues, (Dshalalow, J. H., *Advances in Queueing: Theory, Methods and Open Problems*, (1995), CRC Press Boca Raton), 293-310 · [Zbl 0845.60099](#)
- [27] Dickson, D. C.M.; Hipp, C., On the time to ruin for Erlang(2) risk processes, *Insurance Math. Econom.*, 29, 3, 333-344, (2001) · [Zbl 1074.91549](#)
- [28] Dickson, D. C.M.; Waters, H. R., Some optimal dividends problems, *ASTIN Bull.*, 34, 1, 49-74, (2004) · [Zbl 1097.91040](#)
- [29] Egidio dos Reis, A., How long is the surplus below zero?, *Insurance Math. Econom.*, 12, 1, 23-28, (1993) · [Zbl 0777.62096](#)
- [30] Gerber, H. U., When does the surplus reach a given target?, *Insurance Math. Econom.*, 9, 2, 115-119, (1990) · [Zbl 0731.62153](#)
- [31] Gerber, H. U.; Shiu, E. S.W., On the time value of ruin, *N. Am. Actuar. J.*, 2, 1, 48-72, (1998) · [Zbl 1081.60550](#)
- [32] Gerber, H. U.; Smith, N., Optimal dividends with incomplete information in the dual model, *Insurance Math. Econom.*, 43, 2, 227-233, (2008) · [Zbl 1189.91074](#)
- [33] Grandell, J., *Aspects of risk theory*, (1991), Springer New York · [Zbl 0717.62100](#)
- [34] Janssen, J.; Reinhard, J.-M., Probabilités de ruine pour une classe de modèles de risque semi-markoviens, *ASTIN Bull.*, 15, 2, 123-134, (1985)
- [35] Labbé, C.; Sendov, H. S.; Sendova, K. P., The gerber-shiu function and the generalized cramér-lundberg model, *Appl. Math. Comput.*, 218, 7, 3035-3056, (2011) · [Zbl 1239.91081](#)
- [36] Labbé, C.; Sendova, K. P., The expected discounted penalty function under a risk model with stochastic income, *Appl. Math. Comput.*, 215, 5, 1852-1867, (2009) · [Zbl 1181.91100](#)
- [37] Landriault, D.; Sendova, K. P., A direct approach to a first-passage problem with applications in risk theory, *Stoch. Models*, 27, 3, 388-406, (2011) · [Zbl 1232.91350](#)
- [38] Latouche, G.; Ramaswami, V., *Introduction to matrix analytic methods in stochastic modeling*, (1999), ASA SIAM Philadelphia · [Zbl 0922.60001](#)
- [39] Li, S.; Garrido, J., On ruin for the Erlang(n) risk process, *Insurance Math. Econom.*, 34, 3, 391-408, (2004) · [Zbl 1188.91089](#)
- [40] Li, G.; Luo, J., Upper and lower bounds for the solutions of Markov renewal equations, *Math. Methods Oper. Res.*, 62, 2, 243-253, (2005) · [Zbl 1101.60066](#)
- [41] Mazza, C.; Rullière, D., A link between wave governed random motions and ruin processes, *Insurance Math. Econom.*, 35, 2, 205-222, (2004) · [Zbl 1103.91045](#)
- [42] Miyazawa, M., A Markov renewal approach to the asymptotic decay of the tail probabilities in risk and queueing processes, *Probab. Eng. Inf. Sci.*, 16, 2, 139-150, (2002) · [Zbl 1005.60094](#)
- [43] Neuts, M. F., *Structured stochastic matrices of M/G/1 type and their applications*, (1989), Marcel Dekker New York · [Zbl 0695.60088](#)
- [44] Ng, A. C.Y., On a dual model with a dividend threshold, *Insurance Math. Econom.*, 44, 2, 315-324, (2009) · [Zbl 1163.91441](#)
- [45] Pafumi, G., Discussion of 'on the time value of ruin', *N. Am. Actuar. J.*, 2, 1, 75-76, (1998)
- [46] Reinhard, J.-M., On a class of semi-Markov risk models obtained as classical risk models in a Markovian environment, *ASTIN Bull.*, 14, 1, 23-43, (1984)
- [47] Ren, J., The discounted joint distribution of the surplus prior to ruin and the deficit at ruin in a sparre Andersen model, *N. Am. Actuar. J.*, 11, 3, 128-136, (2007)
- [48] Seal, H. L., *Stochastic theory of a risk business*, (1969), Wiley New York · [Zbl 0196.23501](#)

- [49] Takács, H., Combinatorial methods in the theory of stochastic processes, (1967), Wiley New York · [Zbl 0162.21303](#)
- [50] Temnov, G., Risk processes with random income, *J. Math. Sci.*, 123, 1, 3780-3794, (2004) · [Zbl 1065.91040](#)
- [51] Willmot, G. E., A note on a class of delayed renewal processes, *Insurance Math. Econom.*, 34, 2, 251-257, (2004) · [Zbl 1114.60068](#)
- [52] Willmot, G. E., On the discounted penalty function in the renewal risk model with general interclaim times, *Insurance Math. Econom.*, 41, 1, 17-31, (2007) · [Zbl 1119.91058](#)
- [53] Woo, J.-K., Some remarks on delayed renewal risk models, *ASTIN Bull.*, 40, 1, 199-219, (2010) · [Zbl 1230.91083](#)
- [54] Yang, H.; Zhu, J., Ruin probabilities of a dual Markov-modulated risk model, *Commun. Statist. Theory Methods*, 37, 20, 3298-3307, (2008) · [Zbl 1292.91100](#)
- [55] Wu, Y., Bounds for the ruin probability under a Markovian modulated risk model, *Commun. Statist. Stoch. Models*, 15, 1, 125-136, (1999) · [Zbl 0920.90042](#)
- [56] Zhang, Z.; Yang, H.; Yang, H., On the absolute ruin in a MAP risk model with debit interest, *Adv. Appl. Probab.*, 43, 1, 77-96, (2011) · [Zbl 1229.91171](#)

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