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Real and p-adic Picard-Vessiot fields. (English) Zbl 1344.34096

Math. Ann. 365, No. 1-2, 93-103 (2016).

The authors consider differential modules M over real and p-adic differential fields K (with fields of constants k real closed or p-adic closed). A Picard-Vessiot field (PVF) L for M/K is a field extension of K with the same field of constants k, where L is equipped with a differentiation extending the one of K and there exists an invertible $d \times d$-matrix F ($d =\dim M$) with entries in L satisfying $F' = AF$, as a field L being generated over K by the entries of F. Using results of J.-P. Serre and P. Deligne, the authors obtain a purely algebraic proof of the existence and unicity of PVFs. They treat the inverse problem for real forms of a semisimple group and they give examples illustrating the relations between differential modules, PVFs and real forms of a linear algebraic group.

Reviewer: Vladimir P. Kostov (Nice)

MSC:

12H20 Abstract differential equations
12H25 p-adic differential equations
34M50 Inverse problems (Riemann-Hilbert, inverse differential Galois, etc.) for ordinary differential equations in the complex domain
11E10 Forms over real fields
11R34 Galois cohomology

Keywords:

Picard-Vessiot field; differential module; field of constants; field extension

Full Text: DOI arXiv

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paper as accurately as possible without claiming the completeness or perfect precision of the matching.