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Partial differential equation for evolution of star-shaped reachability domains of differential inclusions. (English) Zbl 1338.93065

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Summary: The problem of reachability for differential inclusions is an active topic in the recent control theory. Its solution provides an insight into the dynamics of an investigated system and also enables one to design synthesizing control strategies under a given optimality criterion. The primary results on reachability were mostly applicable to convex sets, whose dynamics is described through that of their support functions. Those results were further extended to the viability problem and some types of nonlinear systems. However, non-convex sets can arise even in simple bilinear systems. Hence, the issue of nonconvexity in reachability problems requires a more detailed investigation. The present article follows an alternative approach for this cause. It deals with star-shaped reachability sets, describing the evolution of these sets in terms of radial (Minkowski gauge) functions. The derived partial differential equation is then modified to cope with additional state constraints due to on-line measurement observations. Finally, the last section is on designing optimal closed-loop control strategies using radial functions.

MSC:

93B03 Attainable sets, reachability
49K15 Optimality conditions for problems involving ordinary differential equations

Keywords:

reachability sets; differential inclusion; star-shaped sets; radial (gauge) function; viability; optimal control synthesis

Full Text: DOI

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