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Projectable ℓ-groups and algebras of logic: categorical and algebraic connections. (English)

Zbl 1345.06013
J. Pure Appl. Algebra 220, No. 10, 3514-3532 (2016).

Generalized MV algebras, or *GMV algebras* for short, are "simultaneous generalizations of MV algebras to the noncommutative, unbounded and nonintegral case", while *IGMV algebras* are integral GMV algebras. In the paper [*N. Galatos* and *C. Tsinakis*, J. Algebra 283, No. 1, 254-291 (2005; Zbl 1063.06008)] it was proved in fact that "the categories of IGMV algebras and of negative cones of l-groups with a dense nucleus are equivalent".

This paper answers the natural conjecture "that such an equivalence restricts to an equivalence of the subcategories whose objects are the projectable members of these classes of algebras". The authors prove that, indeed, "the categories of projectable IGMV algebras and of negative cones of projectable l-groups with a dense nucleus are equivalent".

Moreover, by adding the Gödel implication to an IGMV algebra, they introduce the notion of *Gödel GMV algebra* – as an algebra $(M, \wedge, \vee, \cdot, \backslash, /, \to, 1)$ of type $(2, 2, 2, 2, 2, 2, 0)$ such that $(M, \wedge, \vee, \cdot, \backslash, /, 1)$ is an IGMV algebra and $(M, \wedge, \vee, \cdot, \to, 1)$ is a Gödel algebra. And they prove that there is an adjunction between the category of Gödel GMV algebras and a certain category.

Reviewer's remarks: The readability of the paper is restricted by the use of the divisions (the "Chinese sticks") $\backslash$, $/$, instead of the implications $\to$, $\rightsquigarrow$ ($y \to z = z/y$ and $y \rightsquigarrow z = y\backslash z$) coming from logic.

Reviewer: Afrodita Iorgulescu (Bucharest)

MSC:

06F15	Ordered groups
06D35	MV-algebras
03B47	Substructural logics (including relevance, entailment, linear logic, Lambek calculus, BCK and BCI logics)
03G10	Logical aspects of lattices and related structures
06B20	Varieties of lattices
06F05	Ordered semigroups and monoids

Keywords:

generalized MV algebras; integral GMV algebras; projectable IGMV algebras; negative cones of projectable l-groups; Gödel GMV algebras; equivalences of categories

Full Text: DOI

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