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A new proof of the sharpness of the phase transition for Bernoulli percolation and the Ising model. (English) Zbl 1342.82026

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Authors' abstract: We provide a new proof of the sharpness of the phase transition for Bernoulli percolation and the Ising model. The proof applies to infinite-range models on arbitrary locally finite transitive infinite graphs. For Bernoulli percolation, we prove finiteness of the susceptibility in the subcritical regime $\beta < \beta_c$, and the mean-field lower bound $\mathbb{P}_\beta[0 \longleftrightarrow \infty] \geq (\beta - \beta_c)/\beta$ for $\beta > \beta_c$. For finite-range models, we also prove that for any $\beta < \beta_c$, the probability of an open path from the origin to distance n decays exponentially fast in n. For the Ising model, we prove finiteness of the susceptibility for $\beta < \beta_c$, and the mean-field lower bound $\langle\sigma_0\rangle_\beta^+ \geq \sqrt{(\beta^2 - \beta_c^2)/\beta^2}$ for $\beta > \beta_c$. For finite-range models, we also prove that the two-point correlation functions decay exponentially fast in the distance for $\beta < \beta_c$.

Reviewer: E. Ahmed (Mansoura)

MSC:

82B20 Lattice systems (Ising, dimer, Potts, etc.) and systems on graphs arising in equilibrium statistical mechanics

Keywords:

Bernoulli percolation; Ising model

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