Merker, Andreas; **Kaiser, Dieter**; **Hermann, Martin**
Numerical bifurcation analysis of the bipedal spring-mass model. (English) Zbl 1335.37056
Physica D 291, 21-30 (2015).

Summary: The spring-mass model and its numerous extensions are currently one of the best candidates for templates of human and animal locomotion. However, with increasing complexity, their applications can become very time-consuming. In this paper, we present an approach that is based on the calculation of bifurcations in the bipedal spring-mass model for walking. Since the bifurcations limit the region of stable walking, locomotion can be studied by computing the corresponding boundaries. Originally, the model was implemented as a hybrid dynamical system. Our new approach consists of the transformation of the series of initial value problems on different intervals into a single boundary value problem. Using this technique, discontinuities can be avoided and sophisticated numerical methods for studying parametrized nonlinear boundary value problems can be applied. Thus, appropriate extended systems are used to compute transcritical and period-doubling bifurcation points as well as turning points. We show that the resulting boundary value problems can be solved by the simple shooting method with sufficient accuracy, making the application of the more extensive multiple shooting superfluous. The proposed approach is fast, robust to numerical perturbations and allows determining complete manifolds of periodic solutions of the original problem.

MSC:

37N05	Dynamical systems in classical and celestial mechanics
92C15	Developmental biology, pattern formation
34C25	Periodic solutions to ordinary differential equations
34C60	Qualitative investigation and simulation of ordinary differential equation models
37M05	Simulation of dynamical systems

Keywords:

spring-mass model; boundary value problem; bifurcation points; turning points; walking

Software:

RWPKV; RWPM; SOCS

Full Text: DOI

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