Kozlowski, A.; Yamaguchi, K.

The homotopy type of spaces of coprime polynomials revisited. (English) Zbl 1387.55012

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Let $n \geq 2$ and $[n] = \{0, 1, \ldots, n-1\}$. For each subset $\sigma = \{i_1, \ldots, i_s\} \subset [n]$, let $L_\sigma \subset \mathbb{C}^n$ denote the coordinate subspace in $\mathbb{C}^n$ defined by $L_\sigma = \{(x_0, x_1, \ldots, x_{n-1}) \in \mathbb{C}^n \mid x_{i_1} = \cdots = x_{i_s} = 0\}$.

Let I be any collection of subsets of $[n]$ such that $|\sigma| \geq 2$ for all $\sigma \in I$. Let $Y_I \subset \mathbb{C}^n$ be the complement of the arrangement of coordinate subspaces defined by $Y_1 = \mathbb{C}^n \backslash L(I)$, where $L(I) = \cup_{\sigma \in I} L_\sigma$. Consider the natural free $\mathbb{C}^*$-action on Y_I given by coordinate-wise multiplication and let X_I denote the orbit space.

For I any collection of subsets of $[n]$ and $(X, *)$ a based space, let $\vee^I X \subset X^n$ denote the subspace consisting of all $(x_0, \ldots, x_{n-1}) \in X^n$ such that, for each $\sigma \in I$, $x_j = *$ for some $j \in \sigma$. The space $\vee^I X$ is called the generalized wedge product of X of type I and there is a homotopy equivalence $\Omega^2_d X_I \simeq \Omega^2(\vee^I \mathbb{C}P^\infty)$.

The purpose of this paper is to study the topology of certain toric varieties X_I and to improve the classical homotopy stability dimension for the inclusion map $i_d : \mathrm{Hol}^*_d(S^2, X_I) \to \mathrm{Map}^*_d(S^2, X_I)$ by making use of the Vassiliev spectral sequence. The authors also improve the homotopy stability dimension of this inclusion given by G. Segal for $X_I = \mathbb{C}P^{n-1}$ and $n \geq 3$.

Let $r_{\min}(I)$ denote the positive integer defined by $r_{\min}(I) = \min\{|\sigma| : \sigma \in I\}$.

The main results are:

a) If $r_{\min}(I) \geq 3$, the inclusion map

$$i_d : \mathrm{Hol}^*_d(S^2, X_I) \to \mathrm{Map}^*_d(S^2, X_I) = \Omega 2_d X_I \simeq \Omega^2(\vee^I \mathbb{C}P^\infty)$$

is a homotopy equivalence through dimension $D(I; d) = (2r_{\min}(I) - 3)d - 2$.

b) (The case $I = I(n)$). If $n \geq 3$, the inclusion map

$$i_d : \mathrm{Hol}^*_d(S^2, \mathbb{C}P^{n-1}) \to \mathrm{Map}^*_d(S^2, \mathbb{C}P^{n-1}) = \Omega 2_d \mathbb{C}P^{n-1} \simeq \Omega^2 S^{2n-1}$$

is a homotopy equivalence through dimension $D^*(d, n) = (2n-3)(d+1) - 1$.

Reviewer: Jelena Grbic (Manchester)

MSC:

55P10 Homotopy equivalences in algebraic topology
55R80 Discriminantal varieties and configuration spaces in algebraic topology
55P35 Loop spaces
14M25 Toric varieties, Newton polyhedra, Okounkov bodies

Keywords:

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