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Neighbor sum distinguishing total choosability of planar graphs without 4-cycles. (English)

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Summary: Let $G = (V, E)$ be a graph and ϕ be a total k-coloring of G by using the color set $\{1, \ldots, k\}$. Let $\sum_\phi(u)$ denote the sum of the color of the vertex u and the colors of all incident edges of u. A k-neighbor sum distinguishing total coloring of G is a total k-coloring of G such that for each edge $uv \in E(G)$, $\sum_\phi(u) \neq \sum_\phi(v)$. By $\chi''_\Sigma(G)$, we denote the smallest value k in such a coloring of G. *M. Pilśniak* and *M. Woźniak* [Graphs Comb. 31, No. 3, 771–782 (2015; Zbl 1312.05054)] first introduced this coloring and conjectured that $\chi''_\Sigma(G) \leq \Delta(G) + 3$ for any simple graph G. Let $L_z(z \in V \cup E)$ be a set of lists of integer numbers, each of size k. The smallest k for which for any specified collection of such lists, there exists a neighbor sum distinguishing total coloring using colors from L_z for each $z \in V \cup E$ is called the neighbor sum distinguishing total choosability of G, and denoted by $\mathrm{ch}''_\Sigma(G)$. In this paper, we prove that $\mathrm{ch}''_\Sigma(G) \leq \Delta(G) + 3$ for planar graphs without 4-cycles with $\Delta(G) \geq 7$. This implies that M. Pilśniak and M. Woźniak's conjecture [loc. cit.] is true for planar graphs without 4-cycles.

MSC:

05C10 Planar graphs; geometric and topological aspects of graph theory
05C15 Coloring of graphs and hypergraphs

Keywords:

neighbor sum distinguishing total coloring; choosability; combinatorial nullstellensatz; planar graph

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