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**Conical limit points and the Cannon-Thurston map.** (English) Zbl 1375.20045  
Conform. Geom. Dyn. 20, 58-80 (2016).

Summary: Let  $G$  be a non-elementary word-hyperbolic group acting as a convergence group on a compact metrizable space  $Z$  so that there exists a continuous  $G$ -equivariant map  $i : \partial G \rightarrow Z$ , which we call a *Cannon-Thurston map*. We obtain two characterizations (a dynamical one and a geometric one) of conical limit points in  $Z$  in terms of their pre-images under the Cannon-Thurston map  $i$ . As an application we prove, under the extra assumption that the action of  $G$  on  $Z$  has no accidental parabolics, that if the map  $i$  is not injective, then there exists a non-conical limit point  $z \in Z$  with  $|i^{-1}(z)| = 1$ . This result applies to most natural contexts where the Cannon-Thurston map is known to exist, including subgroups of word-hyperbolic groups and Kleinian representations of surface groups. As another application, we prove that if  $G$  is a non-elementary torsion-free word-hyperbolic group, then there exists  $x \in \partial G$  such that  $x$  is not a “controlled concentration point” for the action of  $G$  on  $\partial G$ .

**MSC:**

- 20F67 Hyperbolic groups and nonpositively curved groups
- 20F65 Geometric group theory
- 30F40 Kleinian groups (aspects of compact Riemann surfaces and uniformization)
- 37C85 Dynamics induced by group actions other than  $\mathbb{Z}$  and  $\mathbb{R}$ , and  $\mathbb{C}$
- 37F40 Geometric limits in holomorphic dynamics
- 37F30 Quasiconformal methods and Teichmüller theory, etc. (dynamical systems) (MSC2010)
- 57M60 Group actions on manifolds and cell complexes in low dimensions

Cited in 3 Documents

**Keywords:**

convergence groups; Cannon-Thurston map; conical limit points; Kleinian groups

**Full Text:** [DOI](#) [arXiv](#)

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