Schulz-Baldes, Hermann

$\mathbb{Z}_2$-indices and factorization properties of odd symmetric Fredholm operators. (English)

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Summary: A bounded operator T on a separable, complex Hilbert space is said to be odd symmetric if $I^*T^tI = T$ where I is a real unitary satisfying $I^2 = -1$ and T^t denotes the transpose of T. It is proved that such an operator can always be factorized as $T = I^*A^tIA$ with some operator A. This generalizes a result of *L.-K. Hua* [Am. J. Math. 66, 470–488 (1944; Zbl 0063.02919)] and *C. L. Siegel* [ibid. 65, 1–86 (1943; Zbl 0138.31401)] for matrices. As application, it is proved that the set of odd symmetric Fredholm operators has two connected components labelled by a $\mathbb{Z}_2$-index given by the parity of the dimension of the kernel of T. This recovers a result of *M. F. Atiyah* and *I. M. Singer* [Publ. Math., Inst. Hautes Étud. Sci. 37, 5–26 (1969; Zbl 0194.55503)]. Two examples of $\mathbb{Z}_2$-valued index theorems are provided, one being a version of the Noether-Gohberg-Krein theorem with symmetries and the other an application to topological insulators.

MSC:

47A53 (Semi-) Fredholm operators; index theories
81V70 Many-body theory; quantum Hall effect
82D30 Statistical mechanical studies of random media, disordered materials (including liquid crystals and spin glasses)

Keywords:

$\mathbb{Z}_2$-valued index theorems; Noether-Gohberg-Krein theorem with symmetries; topological insulators

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