Let $T$ be a locally compact topological group and let $\mathcal{R}$ be the Banach algebra of bounded right uniformly continuous functions on $T$. Let $\mu$ be a regular Borel probability measure on $T$ ($\mu \in M(T)$). Denote by $|\mathcal{R}|$ the Gelfand space of $\mathcal{R}$. It can be shown that $|\mathcal{R}|$ contains $T$ as a dense subset, $|\mathcal{R}|, T$ is a right transformation group and $|\mathcal{R}|$ has a semi-group structure which induces a semi-group structure on $M(|\mathcal{R}|)$ (the regular Borel probability measures on $|\mathcal{R}|$) via convolution.

A function $f \in \mathcal{R}$ is said to be $\mu$-harmonic if $\int_{|\mathcal{R}|} f(t') \nu(t') \, dt' = f(t)$ ($t \in T$). Let $\mathcal{H}_\mu$ be the set of $\mu$-harmonic functions in $\mathcal{R}$. Let $\mu \in M(T)$. By Kakutani’s fixed point theorem there is an idempotent measure $\nu$ ($\nu^2 = \nu$) on $|\mathcal{R}|$ such that $\nu \cdot \mu = \nu$ and $\mathcal{H}_\nu = \mathcal{H}_\mu$. If the support of $\mu$ is all of $T$, then the support $S$ of $\nu$ is a subflow of $|\mathcal{R}|$ and the restriction $\mathcal{R} : \mathcal{R} \to C(S)$ maps the set $\mathcal{H}_\mu$ of $\mu$-harmonic functions isometrically onto a uniformly closed $T$- invariant subalgebra $\mathcal{H}_\mu$ of $C(S)$. The Poisson boundary of $\mu$ is the Gelfand space of the image of $\mathcal{H}_\mu$ in $C(S)$. It is shown that $B = \{ \nu \cdot \rho | \rho \in S \}$.

This result is used to give elegant proofs of several deep results in boundary theory. For example the only $\mu$-harmonic functions for a symmetric measure $\mu$ absolutely continuous with respect to Haar measure on a solvable Lie group are the constants. Another consequence is that using the structure of the Lie algebra of a connected semi-simple Lie group with finite center and no-compact factors it is shown that $T$ acts transitively on the Poisson boundary of any $\mu \in M(T)$ such that $\mu$ is absolutely continuous and $\sup p(\mu) = T$.

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