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Global solutions to the generalized Leray-alpha equation with mixed dissipation terms. (English) Zbl 1334.76035

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Summary: Due to the intractability of the Navier-Stokes equation, it is common to study approximating equations. Two of the most common of these are the Leray-α equation (which replaces the solution u with $(1-\alpha^2\mathcal{L}_1)u$ for a Fourier Multiplier $\mathcal{L}$) and the generalized Navier-Stokes equation (which replaces the viscosity term $\nu\triangle$ with $\nu\mathcal{L}_2$). In this paper we consider the combination of these two equations, called the generalized Leray-α equation. We provide a brief outline of the typical strategies used to solve such equations, and prove, with initial data in a low-regularity $L^p(\mathbb{R}^n)$ based Sobolev space, the existence of a unique local solution with $\gamma_1+\gamma_2 > n/p+1$. In the $p=2$ case, the local solution is extended to a global solution, improving on previously known results.

MSC:

76D05 Navier-Stokes equations for incompressible viscous fluids
35A01 Existence problems for PDEs: global existence, local existence, non-existence

Cited in **3** Documents

Keywords:

Leray-alpha model; fractional Laplacian; global existence

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