Ahmad, Fayyaz; **Tohidi, Emran**; **Carrasco, Juan A.**

A parameterized multi-step Newton method for solving systems of nonlinear equations. (English) Zbl 1350.65046

Numer. Algorithms 71, No. 3, 631-653 (2016).

The authors introduce a new multi-step method solving systems nonlinear equations $\mathbf{F}(\mathbf{x}) = 0$, where $\mathbf{F} : \Gamma \subseteq \mathbb{R}^n \to \mathbb{R}^r$ is Fréchet differentiable at $\mathbf{x} \in \text{interior}(\Gamma)$ with $\mathbf{F}(\mathbf{x}^*) = 0$ and $\det(\mathbf{F}'(x^*)) \neq 0$. They prove that the method needs m steps to obtain $m + 1$ convergence order. The method is a generalization of the multi-step Newton method based on a parameter θ. Applying the method for solving the nonlinear complex Zakharov system [*A. H. Bhrawy*, Appl. Math. Comput. 247, 30–46 (2014; Zbl 1339.65188)], the authors show that the appropriate choice of θ leads to faster convergence and larger radius of convergence.

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MSC:

65H10 Numerical computation of solutions to systems of equations
65N22 Numerical solution of discretized equations for boundary value problems involving PDEs

Keywords:

multi-step iterative methods; multi-step Newton method; systems of nonlinear equations; discretization methods for partial differential equations; nonlinear complex Zakharov system

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