Lercier, Reynald; Ritzenthaler, Christophe; Sijsling, Jeroen

Explicit Galois obstruction and descent for hyperelliptic curves with tamely cyclic reduced automorphism group. (English) Zbl 1343.14047

Math. Comput. 85, No. 300, 2011-2045 (2016).

Let K be the algebraic closure of a perfect field with $\mathrm{char}(K) \neq 2$, and let F be a subfield of K. Let X be a variety defined over F, and *the field of moduli k of X* is the intersection of all subfields L of K such that there is a variety Y defined over L and $X_K \cong Y_K$. The field of moduli k of X has the property that $X_K \cong X_K^\sigma$ for $\sigma \in \mathrm{Aut}(K)$ if and only if $\sigma \in \mathrm{Gal}(K/k)$. In [Am. J. Math. 78, 509–524 (1956; Zbl 0072.16001)], *A. Weil* asked if there is a model of X over the field of moduli k. Such a model, if exists, is called a *descent* of X, and we say, X has a descent for the extension K/k. Weil showed that if $\mathrm{Aut}(X_K)$ is trivial, then a descent of X exists.

If X is a hyperelliptic curve, the presence of the hyperelliptic involution ι makes $\mathrm{Aut}(X_K)$ nontrivial. If we further require the existence of a model in the form of a hyperelliptic equation $y^2 = p(x)$ over k, we call such a model a *hyperelliptic descent* of X. Let $G = \mathrm{Aut}(X_K)$, and $\overline{G} = G/\langle\iota\rangle$. In [*B. Huggins*, Math. Res. Lett. 14, No. 2, 249–262 (2007; Zbl 1126.14036)], it was shown that X has a hyperelliptic descent if $\overline{G}$ is not cyclic of order coprime to $\mathrm{char}(K)$, and if it is, examples of X with no hyperelliptic descents were constructed in [*C. J. Earle*, in: Adv. Theory Riemann Surfaces, Proc. 1969 Stony Brook Conf., 119–130 (1971; Zbl 0218.32010)] and [*G. Shimura*, Nagoya Math. J. 45, 167–178 (1972; Zbl 0243.14012)]. Recently, found in [*E. Bujalance* and *P. Turbek*, Manuscr. Math. 108, No. 1, 1–11 (2002; Zbl 0997.14008)] is the full classification of hyperelliptic curves that has a hyperelliptic descent for $\mathbb{C}/\mathbb{R}$.

In this paper under review the authors present a complete answer to the descent problem for the case where K/k is any extenstion and $\overline{G}$ is cyclic of order coprime to $\mathrm{char}(K)$. They prove that there is always a field extension L of k with minimal degree $[L : k] \leq 2$ such that X has a hyperelliptic model over L, and they give us explicit conditions on determining when $[L : k] = 1$ or 2. The paper also presents how a descent can be effectively constructed, and given a quadratic extension L/k, it gives an explicit description of the K-isomorphism classes of the curves which are defined over L and K-isomorphic to their conjugates, but do not descend to k.

Reviewer: Sungkon Chang (Savannah)

MSC:

14Q05	Computational aspects of algebraic curves
13A50	Actions of groups on commutative rings; invariant theory
14H10	Families, moduli of curves (algebraic)
14H25	Arithmetic ground fields for curves
14H37	Automorphisms of curves

Keywords:

hyperelliptic curves; Galois descent; field of definition; field of moduli

Software:

Magma

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