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**Linear-space data structures for range frequency queries on arrays and trees.** (English)

Zbl 1411.68034

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Summary: We present  $O(n)$ -space data structures to support various range frequency queries on a given array  $A[0 : n - 1]$  or tree  $T$  with  $n$  nodes. Given a query consisting of an arbitrary pair of pre-order rank indices  $(i, j)$ , our data structures return a least frequent element, mode,  $\alpha$ -minority, or top- $k$  colors (values) of the multiset of elements in the unique path with endpoints at indices  $i$  and  $j$  in  $A$  or  $T$ . We describe a data structure that supports range least frequent element queries on arrays in  $O(\sqrt{n/w})$  time, improving the  $\Theta(\sqrt{n})$  worst-case time required by the data structure of *T. M. Chan* et al. [Lect. Notes Comput. Sci. 7357, 295–306 (2012; Zbl 1318.68068)], where  $w \in \Omega(\log n)$  is the word size in bits. We describe a data structure that supports path mode queries on trees in  $O(\log \log n \sqrt{n/w})$  time, improving the  $\Theta(\sqrt{n} \log n)$  worst-case time required by the data structure of *D. Krizanc* et al. [Lect. Notes Comput. Sci. 2906, 517–526 (2003; Zbl 1205.68130)]. We describe the first data structures to support path least frequent element queries, path  $\alpha$ -minority queries, and path top- $k$  color queries on trees in  $O(\log \log n \sqrt{n/w})$ ,  $O(\alpha^{-1} \log \log n)$ , and  $O(k)$  time, respectively, where  $\alpha \in [0, 1]$  and  $k \in \{1, \dots, n\}$  are specified at query time.

**MSC:**

68P05 Data structures

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data structures; range query; path query; linear space; frequency; color query

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