

Coletti, Giulianella; Petturiti, Davide

Finitely maxitive T -conditional possibility theory: coherence and extension. (English)

Zbl 1352.68241

Int. J. Approx. Reasoning 71, 64-88 (2016).

Summary: Starting from the axiomatic definition of finitely maxitive T -conditional possibility (where T is a continuous triangular norm), the paper aims at a comprehensive and self-contained treatment of coherence and extension of a possibilistic assessment defined on an arbitrary set of conditional events. Coherence (or consistence with a T -conditional possibility) is characterized either in terms of existence of a linearly ordered class of finitely maxitive possibility measures (T -nested class) agreeing with the assessment, or in terms of solvability of a finite sequence of nonlinear systems for every finite subfamily of conditional events. Coherence reveals to be a necessary and sufficient condition for the extendibility of an assessment to any superset of conditional events and, in the case of T equal to the minimum or a strict t-norm, the set of coherent values for the possibility of a new conditional event can be computed solving two optimization problems over a finite sequence of nonlinear systems for every finite subfamily of conditional events.

MSC:

68T37 Reasoning under uncertainty in the context of artificial intelligence

Cited in 2 Documents

Keywords:

finitely maxitive measure; T -conditional possibility; coherence; extension

Full Text: [DOI](#)

References:

- [1] Acerbi, E.; Buttazzo, G.; Prinari, F., The class of functionals which can be represented by a supremum, *J. Convex Anal.*, 9, 1, 225-236, (2002) · Zbl 1012.49010
- [2] Akian, M., Densities of idempotent measures and large deviations, *Trans. Am. Math. Soc.*, 35, 11, 4515-4543, (1999) · Zbl 0934.28005
- [3] Allamigeon, X.; Gaubert, S.; Katz, R. D., Tropical polar cones, hypergraph transversals, and mean payoff games, *Linear Algebra Appl.*, 435, 7, 1549-1574, (2011) · Zbl 1217.14047
- [4] Armstrong, T. E., Countably additive full conditional probabilities, *Proc. Am. Math. Soc.*, 107, 977-987, (1989) · Zbl 0689.28002
- [5] Biaoletti, M.; Coletti, G.; Petturiti, D.; Vantaggi, B., Inferential models and relevant algorithms in a possibilistic framework, *Int. J. Approx. Reason.*, 52, 5, 580-598, (2011) · Zbl 1214.68393
- [6] Biaoletti, M.; Petturiti, D., Algorithms for possibility assessments: coherence and extension, *Fuzzy Sets Syst.*, 169, 1, 1-25, (2011) · Zbl 1214.68394
- [7] Bhaskara Rao, K. P.S.; Bhaskara Rao, M., *Theory of charges: A study of finitely additive measures*, (1983), Academic Press · Zbl 0516.28001
- [8] Bouchon, B., Fuzzy inferences and conditional possibility distributions, *Fuzzy Sets Syst.*, 23, 1, 33-41, (1987) · Zbl 0633.68100
- [9] Bouchon-Meunier, B.; Coletti, G.; Marsala, C., Conditional possibility and necessity, (Bouchon-Meunier, B.; Gutiérrez-Ríos, J.; Magdalena, L.; Yager, R. R., *Technologies for Constructing Intelligent Systems 2*, Studies in Fuzziness and Soft Computing, vol. 90, (2002), Physica-Verlag HD), 59-71 · Zbl 1015.68191
- [10] Bouchon-Meunier, B.; Coletti, G.; Marsala, C., Independence and possibilistic conditioning, *Ann. Math. Artif. Intell.*, 35, 1-4, 107-123, (2002) · Zbl 1004.60001
- [11] Busanello, G.; Petturiti, D., DAG representation of asymmetric independence models arising in coherent conditional possibility theory, *Fuzzy Sets Syst.*, 250, 1-24, (2014) · Zbl 1334.68216
- [12] Coletti, G., Coherent numerical and ordinal probabilistic assessments, *IEEE Trans. Syst. Man Cybern.*, 24, 12, 1747-1754, (1994) · Zbl 1371.68265
- [13] Coletti, G.; Petturiti, D., Bayesian-like inference, complete disintegrability and complete conglomerability in coherent conditional possibility theory, (Cozman, F.; Dencøux, T.; Destercke, S.; Seidenfeld, T., *ISIPTA'13: Proceedings of the Eighth International Symposium on Imprecise Probability: Theories and Applications*, Compiègne, (2013), SIPTA), 55-65

- [14] Coletti, G.; Petturiti, D., Finitely maxitive conditional possibilities, Bayesian-like inference, disintegrability and conglomerability, *Fuzzy Sets Syst.*, 284, 31-55, (2016) · [Zbl 1383.62060](#)
- [15] Coletti, G.; Petturiti, D.; Vantaggi, B., Independence in possibility theory under different triangular norms, (van der Gaag, Linda L. C., *Symbolic and Quantitative Approaches to Reasoning with Uncertainty*, Lecture Notes in Computer Science, vol. 7958, (2013), Springer Berlin, Heidelberg), 133-144 · [Zbl 1390.68642](#)
- [16] Coletti, G.; Petturiti, D.; Vantaggi, B., Coherent T-conditional possibility envelopes and nonmonotonic reasoning, (Laurent, A.; Strauss, O.; Bouchon-Meunier, B.; Yager, R. R., *Information Processing and Management of Uncertainty in Knowledge-Based Systems*, Communications in Computer and Information Science, vol. 444, (2014), Springer International Publishing), 446-455
- [17] Coletti, G.; Petturiti, D.; Vantaggi, B., Possibilistic and probabilistic likelihood functions and their extensions: common features and specific characteristics, *Fuzzy Sets Syst.*, 250, 25-51, (2014) · [Zbl 1334.60004](#)
- [18] Coletti, G.; Petturiti, D.; Vantaggi, B., Conditional belief functions as lower envelopes of conditional probabilities in a finite setting, *Inf. Sci.*, 339, 64-84, (2016) · [Zbl 1395.68261](#)
- [19] Coletti, G.; Scozzafava, R., Characterization of coherent conditional probabilities as a tool for their assessment and extension, *Int. J. Uncertain. Fuzziness Knowl.-Based Syst.*, 4, 2, 103-127, (1996) · [Zbl 1232.03010](#)
- [20] Coletti, G.; Scozzafava, R., Conditioning and inference in intelligent systems, *Soft Comput.*, 3, 3, 118-130, (1999)
- [21] Coletti, G.; Scozzafava, R., From conditional events to conditional measures: a new axiomatic approach, *Ann. Math. Artif. Intell.*, 32, 1-4, 373-392, (2001) · [Zbl 1314.68306](#)
- [22] Coletti, G.; Scozzafava, R., Probabilistic logic in a coherent setting, *Trends in Logic*, vol. 15, (2002), Kluwer Academic Publishers Dordrecht/Boston/London · [Zbl 1005.60007](#)
- [23] Coletti, G.; Scozzafava, R., Coherent conditional probability as a measure of uncertainty of the relevant conditioning events, (*Symbolic and Quantitative Approaches to Reasoning with Uncertainty*, Lecture Notes in Artificial Intelligence, vol. 2711, (2003), Springer Berlin, Heidelberg), 407-418 · [Zbl 1274.68520](#)
- [24] Coletti, G.; Scozzafava, R., Conditional probability, fuzzy sets, and possibility: a unifying view, *Fuzzy Sets Syst.*, 144, 1, 227-249, (2004) · [Zbl 1076.68082](#)
- [25] Coletti, G.; Scozzafava, R.; Vantaggi, B., Integrated likelihood in a finitely additive setting, (Sossai, C.; Chemello, G., *Symbolic and Quantitative Approaches to Reasoning with Uncertainty*, Lecture Notes in Computer Science, vol. 5590, (2009), Springer Berlin, Heidelberg), 554-565 · [Zbl 1245.62012](#)
- [26] Coletti, G.; Scozzafava, R.; Vantaggi, B., Inferential processes leading to possibility and necessity, *Inf. Sci.*, 245, 132-145, (2013) · [Zbl 1320.68181](#)
- [27] Coletti, G.; Vantaggi, B., Possibility theory: conditional independence, *Fuzzy Sets Syst.*, 157, 11, 1491-1513, (2006) · [Zbl 1092.68094](#)
- [28] Coletti, G.; Vantaggi, B., A view on conditional measures through local representability of binary relations, *Int. J. Approx. Reason.*, 47, 3, 268-283, (2008) · [Zbl 1184.68500](#)
- [29] Coletti, G.; Vantaggi, B., T-conditional possibilities: coherence and inference, *Fuzzy Sets Syst.*, 160, 3, 306-324, (2009) · [Zbl 1178.60006](#)
- [30] de Cooman, G., Possibility theory I: the measure- and integral-theoretic groundwork, *Int. J. Gen. Syst.*, 25, 291-323, (1997) · [Zbl 0955.28012](#)
- [31] de Cooman, G., Possibility theory II: conditional possibility, *Int. J. Gen. Syst.*, 25, 4, 325-351, (1997) · [Zbl 0955.28013](#)
- [32] de Cooman, G., Integration and conditioning in numerical possibility theory, *Ann. Math. Artif. Intell.*, 32, 1-4, 87-123, (2001) · [Zbl 1314.28012](#)
- [33] de Cooman, G.; Zhang, G.; Kerre, E. E., Possibility measures and possibility integrals defined on a complete lattice, *Fuzzy Sets Syst.*, 120, 3, 459-467, (2001) · [Zbl 1017.28010](#)
- [34] de Finetti, B.; de Finetti, B., Aggiunta alla nota sull'assiomatica Della probabilità, *Ann. Triest.*, *Ann. Triest.*, 20, 3-20, (1949) · [Zbl 0044.13402](#)
- [35] de Finetti, B., *Probability, induction and statistics: the art of guessing*, (1972), John Wiley & Sons London, New York, Sydney, Toronto · [Zbl 0275.60001](#)
- [36] Dempster, A. P., Upper and lower probabilities induced by a multivalued mapping, *Ann. Math. Stat.*, 38, 2, 325-339, (1967) · [Zbl 0168.17501](#)
- [37] Dubins, L. E., Finitely additive conditional probabilities, conglomerability and disintegrations, *Ann. Probab.*, 3, 1, 89-99, (1975) · [Zbl 0302.60002](#)
- [38] Dubois, D., Belief structure, possibility theory and decomposable confidence measures on finite sets, *Comput. Artif. Intell.*, 5, 5, (1986) · [Zbl 0657.60006](#)
- [39] Dubois, D.; Prade, H., *Possibility theory: an approach to computerized processing of uncertainty*, (1988), Plenum Press New York and London
- [40] Dubois, D.; Prade, H., When upper probabilities are possibility measures, *Fuzzy Sets Syst.*, 49, 1, 65-74, (1992) · [Zbl 0754.60003](#)
- [41] Dubois, D.; Prade, H., Bayesian conditioning in possibility theory, *Fuzzy Sets Syst.*, 92, 2, 223-240, (1997) · [Zbl 1053.62503](#)
- [42] Dubois, D.; Prade, H., Possibility theory: qualitative and quantitative aspects, (Smets, P., *Quantified Representation of Uncertainty and Imprecision*, Handbook of Defeasible Reasoning and Uncertainty Management Systems, vol. 1, (1998), Springer Netherlands), 169-226 · [Zbl 0924.68182](#)

- [43] El Rayes, A. B.; Morsi, N. N., Generalized possibility measures, *Inf. Sci.*, 79, 3-4, 201-222, (1994) · [Zbl 0812.28012](#)
- [44] Ferracuti, L.; Vantaggi, B., Independence and conditional possibility for strictly monotone triangular norms: research articles, *Int. J. Intell. Syst.*, 21, 3, 299-323, (2006) · [Zbl 1088.60003](#)
- [45] Gaubert, S.; Katz, R. D., The tropical analogue of polar cones, *Linear Algebra Appl.*, 431, 5-7, 608-625, (2009) · [Zbl 1172.52002](#)
- [46] Hisdal, E., Conditional possibilities independence and noninteraction, *Fuzzy Sets Syst.*, 1, 4, 283-297, (1978) · [Zbl 0393.94050](#)
- [47] Holzer, S., Sulla nozione di coerenza per le probabilità subordinate, *Rend. Ist. Mat. Univ. Trieste*, 16, 46-62, (1984) · [Zbl 0656.60008](#)
- [48] Klement, E. P.; Mesiar, R.; Pap, E., *Triangular norms*, vol. 8, (2000), Kluwer Academic Publishers Dordrecht/Boston/London · [Zbl 0972.03002](#)
- [49] Krauss, P. H., Representation of conditional probability measures on Boolean algebras, *Acta Math. Acad. Sci. Hung.*, 19, 3-4, 229-241, (1968) · [Zbl 0174.49001](#)
- [50] Maslov, V. P., *Méthodes opératorielles*, (1987), Éditions Mir Moscow · [Zbl 0651.47039](#)
- [51] Miranda, E.; Couso, I.; Gil, P., Relationships between possibility measures and nested random sets, *Int. J. Uncertain. Fuzziness Knowl.-Based Syst.*, 10, 01, 1-15, (2002) · [Zbl 1084.68129](#)
- [52] Miranda, E.; Couso, I.; Gil, P., A random set characterization of possibility measures, *Inf. Sci.*, 168, 1-4, 51-75, (2004) · [Zbl 1074.28010](#)
- [53] Nguyen, H. T., Fuzzy sets as a basis for a theory of possibility, *Fuzzy Sets Syst.*, 1, 2, 299-309, (1978)
- [54] Nguyen, H. T.; Bouchon-Meunier, B., Random sets and large deviations principle as a foundation for possibility measures, *Soft Comput.*, 8, 61-70, (2003) · [Zbl 1155.68522](#)
- [55] Petturiti, D., Asymmetric decomposability and persegram representation in coherent conditional probability theory, *Soft Comput.*, 17, 11, 2131-2145, (2013) · [Zbl 1410.68354](#)
- [56] Petturiti, D., Coherent conditional possibility theory and possibilistic graphical modeling in a coherent setting, (2013), Università degli Studi di Perugia Perugia, Italy, PhD thesis
- [57] Poncet, P., Infinite-dimensional idempotent analysis: the role of continuous posets, (2012), École Polytechnique, ParisTech Paris, France, PhD thesis
- [58] Puhalskii, A., Large deviations and idempotent probability, *Monographs and Surveys in Pure and Applied Mathematics*, vol. 119, (2001), CRC Press · [Zbl 0983.60003](#)
- [59] Regazzini, E., De Finetti's coherence and statistical inference, *Ann. Stat.*, 15, 2, 845-864, (1987) · [Zbl 0653.62003](#)
- [60] Rényi, A., On conditional probability spaces generated by a dimensionally ordered set of measures, *Theory Probab. Appl.*, 1, 61-71, (1956) · [Zbl 0073.12302](#)
- [61] Shilkret, N., Maxitive measure and integration, *Indag. Math. (Proc.)*, 74, 109-116, (1971) · [Zbl 0218.28005](#)
- [62] Singpurwalla, N. D.; Booker, J. M., Membership functions and probability measures of fuzzy sets, *J. Am. Stat. Assoc.*, 99, 467, 867-889, (2004) · [Zbl 1117.62425](#)
- [63] Sugeno, M., *Theory of fuzzy integrals and its applications*, (1974), Tokyo Institute of Technology Tokyo, Japan, PhD thesis
- [64] Walley, P., Coherent lower (and upper) probabilities, (1981), Department of Statistics, University of Warwick, Technical report
- [65] Willard, S., *General topology*, (1970), Addison-Wesley Publishing Company Reading, Massachusetts · [Zbl 0205.26601](#)
- [66] Williams, P. M., Note on conditional previsions, *Int. J. Approx. Reason.*, 44, 366-383, (2007), School of Mathematical and Physical Science, University of Sussex, published in · [Zbl 1114.60005](#)
- [67] Zadeh, L. A., Fuzzy sets as a basis for a theory of possibility, *Fuzzy Sets Syst.*, 1, 1, 3-28, (1978) · [Zbl 0377.04002](#)

This reference list is based on information provided by the publisher or from digital mathematics libraries. Its items are heuristically matched to zbMATH identifiers and may contain data conversion errors. It attempts to reflect the references listed in the original paper as accurately as possible without claiming the completeness or perfect precision of the matching.