Han, Xiaolong

Let $\Delta$ be the scalar Laplacian of a smooth compact Riemannian manifold $(M, g)$ of dimension $n$ without boundary. Let $\{u_j\}$ for $j = 1, 2, \ldots$ be the normalized $L^2$ eigenvalues so $\Delta u_j = \lambda_j^2 u_j$, where $0 = \lambda_0 < \lambda_1 < \ldots$. One knows (see C. D. Sogge [Duke Math. J. 53, 43-65 (1986; Zbl 0636.42018); J. Funct. Anal. 77, No. 1, 123–138 (1988; Zbl 0641.46011)]) that $\|u_j\|_p \lesssim \lambda_j^{\sigma(p)}$ for suitably chosen $\sigma(p)$. Take $(M, g)$ to be the standard sphere with the round metric. Let $\mathbb{H}_k$ be the set of homogeneous harmonic polynomials of degree $k$; these restrict to eigenfunctions on the sphere with associated eigenvalue $k(k+1)$; the multiplicity of this eigenvalue is given by $\dim(\mathbb{H}_k) = 2k + 1$ and every eigenfunction arises in this way. In relation to Sogge’s estimate, the authors show the following theorem.

Theorem. There exists a constant $0 < D < 1$ so that, for all $k \geq 0$, if $m$ is the greatest integer not greater than $D(2k + 1)$, then one can find an orthonormal set $\{u_i\}$ for $1 \leq i \leq m$ in $\mathbb{H}_k$ such that $\|u_i\|_p \geq \frac{1}{2} C_p k^{\sigma(p)}$ for all $i$ and for $2 < p \leq 6$. For example, $D = \frac{1}{3m}$ will do.

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MSC:

35P20 Asymptotic distributions of eigenvalues in context of PDEs
33C55 Spherical harmonics
58J50 Spectral problems; spectral geometry; scattering theory on manifolds

Keywords:
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References:

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