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**Global well-posedness of the 2D Boussinesq equations with fractional Laplacian dissipation.**

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In this article, the authors consider the Cauchy problem for the 2D incompressible Boussinesq system with fractional Laplacian dissipation. They prove global regularity for the solution with a new range of fractional powers.

Reviewer: [Bernard Ducomet \(Bruyères le Châtel\)](#)

**MSC:**

- [35Q35](#) PDEs in connection with fluid mechanics
- [35B65](#) Smoothness and regularity of solutions to PDEs
- [76D03](#) Existence, uniqueness, and regularity theory for incompressible viscous fluids
- [26A33](#) Fractional derivatives and integrals

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**Keywords:**

[2D Boussinesq equations](#); [fractional Laplacian dissipation](#); [global regularity](#)

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