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**The Conrad program: from  $l$ -groups to algebras of logic.** (English) Zbl 1337.06006  
J. Algebra 450, 173-203 (2016).

Lattice-ordered groups ( $l$ -groups for short) have a fundamental role in the study of algebras of logic. The term Conrad program refers to Paul Conrad's approach to  $l$ -groups focusing on lattice theoretic properties of their lattices of convex  $l$ -subgroups.

In the paper under review the authors develop a Conrad type approach to the study of a large class of algebras of logic, showing that a substantial part of the Conrad program can be extended to residuated lattices that satisfy the identity  $x \setminus e \approx e/x$ , called  $e$ -cyclic in the present paper. This variety encompasses several varieties of significance in algebraic logic. For any  $e$ -cyclic residuated lattice  $L$  let  $C(L)$  be the lattice of its convex subalgebras. A first result proved in this paper is that  $C(L)$  is an algebraic distributive lattice whose principal convex subalgebras form a sublattice.

A second result is that, in case  $L$  satisfies the left or right prelinearity law, a convex subalgebra  $H$  of  $L$  is prime iff the set of all convex subalgebras exceeding  $H$  is a chain under set-inclusion. Thus the lattice of principal convex subalgebras of  $L$  is a relatively normal lattice. It is also proved that a variety  $V$  of  $e$ -cyclic residuated lattices that satisfy either of the prelinearity laws is semilinear iff for every  $L$  in  $V$  all minimal prime convex subalgebras of  $L$  are normal.

In the final part of the paper the authors consider special classes of residuated lattices, e.g., residuated lattices in which convex subalgebras are normal, and GMV-algebras.

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**MSC:**

- [06D35](#) MV-algebras
- [06F05](#) Ordered semigroups and monoids
- [06F15](#) Ordered groups
- [03G10](#) Logical aspects of lattices and related structures
- [03B47](#) Substructural logics (including relevance, entailment, linear logic, Lambek calculus, BCK and BCI logics)
- [08B15](#) Lattices of varieties

Cited in 7 Documents

**Keywords:**

[residuated lattices](#); [Conrad program](#);  [\$l\$ -groups](#); [MV-algebras](#); [GMV-algebras](#)

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