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**The generalized Hill model: a kinematic approach towards active muscle contraction.** (English) [Zbl 1328.74063](#)

*J. Mech. Phys. Solids* 72, 20-39 (2014).

Summary: Excitation-contraction coupling is the physiological process of converting an electrical stimulus into a mechanical response. In muscle, the electrical stimulus is an action potential and the mechanical response is active contraction. The classical Hill model characterizes muscle contraction through one contractile element, activated by electrical excitation, and two non-linear springs, one in series and one in parallel. This rheology translates into an additive decomposition of the total stress into a passive and an active part. Here we supplement this additive decomposition of the stress by a multiplicative decomposition of the deformation gradient into a passive and an active part. We generalize the one-dimensional Hill model to the three-dimensional setting and constitutively define the passive stress as a function of the total deformation gradient and the active stress as a function of both the total deformation gradient and its active part. We show that this novel approach combines the features of both the classical stress-based Hill model and the recent active-strain models. While the notion of active stress is rather phenomenological in nature, active strain is micro-structurally motivated, physically measurable, and straightforward to calibrate. We demonstrate that our model is capable of simulating excitation-contraction coupling in cardiac muscle with its characteristic features of wall thickening, apical lift, and ventricular torsion.

**MSC:**

[74L15](#) Biomechanical solid mechanics

[92C10](#) Biomechanics

[74F15](#) Electromagnetic effects in solid mechanics

[74S05](#) Finite element methods applied to problems in solid mechanics

Cited in **1** Review  
Cited in **11** Documents

**Keywords:**

[coupled cardiac electromechanics](#); [excitation-contraction](#); [active-strain](#); [finite elements](#); [Hill model](#)

**Full Text:** [DOI](#)

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