Żaba, Mariusz; Garbaczewski, Piotr

Nonlocally induced (fractional) bound states: shape analysis in the infinite Cauchy well. (English) Zbl 1332.34133

J. Math. Phys. 56, No. 12, 123502, 21 p. (2015).

Summary: Fractional (Lévy-type) operators are known to be spatially nonlocal. This becomes an issue if confronted with *a priori* imposed exterior Dirichlet boundary data. We address spectral properties of the prototype example of the Cauchy operator $(-\Delta)^{1/2}$ in the interval $D = (-1, 1) \subset R$, with a focus on functional shapes of first few eigenfunctions and their fall-off at the boundary of D. New high accuracy formulas are deduced for approximate eigenfunctions. We analyze how their shape reproduction fidelity is correlated with the evaluation finesse of the corresponding eigenvalues.

MSC:

34L10 Eigenfunctions, eigenfunction expansions, completeness of eigenfunctions of ordinary differential operators
34A08 Fractional ordinary differential equations and fractional differential inclusions
34B10 Nonlocal and multipoint boundary value problems for ordinary differential equations
34D15 Singular perturbations of ordinary differential equations
34L40 Particular ordinary differential operators (Dirac, one-dimensional Schrödinger, etc.)

Full Text: DOI arXiv

References:

[1] Garbaczewski, P.; Stephanovich, V., Lévy flights and nonlocal quantum dynamics, J. Math. Phys., 54, 072103, (2013) · Zbl 1284.81012
[2] Kwaśnicki, M., "Ten equivalent definitions of the fractional Laplace operator," e-print [math.AP]. · Zbl 1375.47038
[3] Zoia, A.; Rosso, A.; Kardar, M., Fractional Laplacian in a bounded domain, Phys. Rev. E, 76, 021116, (2007)
[4] Jeng, M., On the nonlocality of the fractional Schrödinger equation, J. Math. Phys., 51, 062102, (2010) · Zbl 1311.81114
[5] Luchko, Y., Fractional Schrödinger equation for a particle moving in a potential well, J. Math. Phys., 54, 012111, (2013) · Zbl 1280.81044
[6] Getoor, R. K., First pasaage times for symmetric stable processes in space, Trans. Am. Math. Soc., 101, 75, (1961) · Zbl 0104.11203
[7] Garbaczewski, P.; Stephanovich, V., Lévy flights in inhomogeneous environments, Physica A, 389, 4419, (2010)
[8] Lőrinczi, J.; Małecki, J., Spectral properties of the massless relativistic harmonic oscillator, J. Differ. Equations, 251, 2846, (2012) · Zbl 1272.47063
[9] Kwaśnicki, M., Eigenvalues of the fractional Laplace operator in the interval, J. Funct. Anal., 262, 2379, (2012) · Zbl 1234.35164
[10] Bañuelos, R.; Kulczycki, T., The Cauchy process and the Steklov problem, J. Funct. Anal., 211, 355-423, (2004) · Zbl 1055.60072
[11] Bañuelos, R.; Kulczycki, T.; Méndez-Hernández, P. J., On the shape of the ground state eigenfunction for stable processes, Potential Anal., 24, 205-221, (2006) · Zbl 1105.31006
[12] Kulczycki, T.; Kwaśnicki, M.; Małecki, J.; Stós, A., Spectral properties of the Cauchy process on half-line and interval, Proc. London Math. Soc., 101, 589-622, (2010) · Zbl 1220.60029
[13] Dyda, B., Fractional calculus form power functions and eigenvalues of the fractional Laplacian, Fractional Calculus Appl. Anal., 15, 4, 536, (2012) · Zbl 1312.35176
[14] Dyda, B., Fractional Hardy inequality with a remainder term, Colloq. Math., 122, 1, 59, (2011) · Zbl 1228.26022
[15] Guan, Q.-Y.; Ma, Z.-M., Reflected symmetric α-stable processes and regional fractional Laplacian, Probab. Theory Relat. Fields, 134, 649, (2006) · Zbl 1089.60030

[16] Żaba, M.; Garbaczewski, P., Solving fractional Schrödinger-type spectral problems: Cauchy oscillator and Cauchy well, J. Math. Phys., 55, 092103, (2014) · Zbl 1304.81069

[17] Garbaczewski, P.; Olkiewicz, R., Cauchy noise and affiliated stochastic processes, J. Math. Phys., 40, 1057, (1999) · Zbl 0958.60070

[18] Stein, E. M., Singular Integrals and Differentiability Properties of Functions, (1970), Princeton University Press: Princeton University Press, Princeton · Zbl 0207.13501

[19] Garbaczewski, P.; Karwowski, W., Impenetrable barriers and canonical quantization, Am. J. Phys., 72, 924, (2004)

[20] Cohen-Tannoudji, C.; Diu, B.; Laloë, F., Quantum Mechanics, 1, (1977), Wiley: Wiley, New York

[21] Belloni, M.; Robinett, R. W., The infnite well and Dirac delta function potentials as pedagogical, mathematical and physical models in quantum mechanics, Phys. Rep., 540, 25, (2014) · Zbl 1357.81080

[22] Kwaśnicki, M., private communication (2015).

[23] Kaleta, K.; Kwaśnicki, M.; Małecki, J., One-dimensional quasi-relativistic particle in the box, Rev. Math. Phys., 25, 8, 1350014, (2013) · Zbl 1278.81068

[24] Gradshteyn, I. S.; Ryzhik, I. M.; Zwillinger, D.; Moll, V., Table of Integrals, Series, and Products, (2014), Academic Press

