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Global dynamics of a delayed within-host viral infection model with both virus-to-cell and cell-to-cell transmissions. (English) Zbl 1331.34160

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Summary: A within-host viral infection model with both virus-to-cell and cell-to-cell transmissions and three distributed delays is investigated, in which the first distributed delay describes the intracellular latency for the virus-to-cell infection, the second delay represents the intracellular latency for the cell-to-cell infection, and the third delay describes the time period that viruses penetrated into cells and infected cells release new virions. The global stability analysis of the model is carried out in terms of the basic reproduction number $\mathcal{R}_0$. If $\mathcal{R}_0 \leq 1$, the infection-free (semi-trivial) equilibrium is the unique equilibrium and is globally stable; if $\mathcal{R}_0 > 1$, the chronic infection (positive) equilibrium exists and is globally stable under certain assumptions. Examples and numerical simulations for several special cases are presented, including various within-host dynamics models with discrete or distributed delays that have been well-studied in the literature. It is found that the global stability of the chronic infection equilibrium might change in some special cases when the assumptions do not hold. The results show that the model can be applied to describe the within-host dynamics of HBV, HIV, or HTLV-1 infection.

MSC:

34K60 Qualitative investigation and simulation of models involving functional-differential equations
34K20 Stability theory of functional-differential equations
34K25 Asymptotic theory of functional-differential equations
92C60 Medical epidemiology
34K18 Bifurcation theory of functional-differential equations
34K13 Periodic solutions to functional-differential equations

Keywords:

within-host dynamics; distributed delay; global stability; Lyapunov functional; Hopf bifurcation

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