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On the neighbor sum distinguishing total coloring of planar graphs. (English) Zbl 1331.05084
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Summary: Let c be a proper total coloring of a graph $G = (V, E)$ with integers $1, 2, \dots, k$. For any vertex $v \in V(G)$, let $\sum_c(v)$ denote the sum of colors of the edges incident with v and the color of v . If for each edge $uv \in E(G)$, $\sum_c(u) \neq \sum_c(v)$, then such a total coloring is said to be neighbor sum distinguishing. The least k for which such a coloring of G exists is called the neighbor sum distinguishing total chromatic number and denoted by $\chi''_\Sigma(G)$. *M. Piłśniak* and *M. Woźniak* [Graphs Comb. 31, No. 3, 771–782 (2015; Zbl 1312.05054)] conjectured $\chi''_\Sigma(G) \leq \Delta(G) + 3$ for any simple graph with maximum degree $\Delta(G)$. It is known that this conjecture holds for any planar graph with $\Delta(G) \geq 13$. In this paper, we prove that for any planar graph, $\chi''_\Sigma(G) \leq \max\{\Delta(G) + 3, 14\}$.

MSC:

05C15 Coloring of graphs and hypergraphs

05C10 Planar graphs; geometric and topological aspects of graph theory

Cited in **27** Documents

Keywords:

neighbor sum distinguishing total coloring; planar graph; total coloring; discharging; combinatorial nullstellensatz

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