

**Balbi, Valentina; Ciarletta, Pasquale**

**Helical buckling of thick-walled, pre-stressed, cylindrical tubes under a finite torsion.** (English) [Zbl 1327.74063](#)

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**Summary:** We study the occurrence of torsional instabilities in soft, incompressible, thick-walled tubes with both circumferential and axial pre-stretches. Assuming a neo-Hookean strain energy function, we investigate the helical buckling under a finite torsion in three different classes of boundary conditions: (a) no applied loads at the internal and external surfaces of the cylindrical tube, (b) a pressure load  $P$  acting on the external surface or (c) on the internal surface. We perform a linear stability analysis on the axisymmetric solutions using the method of small deformations superposed on finite strains. Applying a helical perturbation, we derive the Stroh formulation of the incremental boundary value problems and we solve it using a numerical procedure based on the surface impedance method. The threshold values of the torsion rate and the associated critical circumferential and longitudinal modes at the onset of the instability are examined in terms of the circumferential and axial pre-stretches, and of the initial geometry of the soft tube.

**MSC:**

[74G60](#) Bifurcation and buckling

[74B20](#) Nonlinear elasticity

**Keywords:**

[finite torsion](#); [elastic stability](#); [residual stresses](#); [Stroh formulation](#); [surface impedance method](#)

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