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Logit dynamics with concurrent updates for local interaction potential games. (English)

Zbl 1338.91035
Algorithmica 73, No. 3, 511-546 (2015).

Summary: Logit choice dynamics constitute a family of randomized best response dynamics based on the logit choice function [*D. McFadden*, "Conditional logit analysis of qualitative choice behavior", in: Frontiers in econometrics. New York, NY: Academic Press. 105–142 (1974)] that models players with limited rationality and knowledge. In this paper we study the all-logit dynamics (also known as simultaneous learning [*C. Alós-Ferrer* and *N. Netzer*, Games Econ. Behav. 68, No. 2, 413–427 (2010; Zbl 1207.91017)]), where at *each* time step *all* players *concurrently* update their strategies according to the logit choice function. In the well studied (one-)logit dynamics [*L. E. Blume*, Games Econ. Behav. 5, No. 3, 387–424 (1993; Zbl 0797.90123)] instead at each step *only one* randomly chosen player is allowed to update. We study properties of the all-logit dynamics in the context of local interaction potential games, a class of games that has been used to model complex social phenomena [*A. Montanari* and *A. Saberi*, in: Proceedings of the 50th annual IEEE symposium on foundations of computer science, FOCS'09. Los Alamitos, CA: IEEE Computer Society. 303–312 (2009; Zbl 1292.91036); *H. Peyton Young*, "The diffusion of innovations in social networks", in: L. E. Blume (ed.) et al., The economy as a complex evolving system. Vol. III. Oxford: Oxford University Press. 267–281 (2003)] and physical systems [*D. A. Levin* et al., Probab. Theory Relat. Fields 146, No. 1–2, 223–265 (2010; Zbl 1187.82076); *F. Martinelli*, Lect. Notes Math. 1717, 93–191 (1999; Zbl 1051.82514)]. In a local interaction potential game players are the vertices of a social graph whose edges are two-player potential games. Each player picks one strategy to be played for all the games she is involved in and the payoff of the player is the sum of the payoffs from each of the games. We prove that local interaction potential games characterize the class of games for which the all-logit dynamics is reversible. We then compare the stationary behavior of one-logit and all-logit dynamics. Specifically, we look at the expected value of a notable class of observables, that we call decomposable observables. We prove that the difference between the expected values of the observables at stationarity for the two dynamics depends only on the rationality level β and on the distance of the social graph from a bipartite graph. In particular, if the social graph is bipartite then decomposable observables have the same expected value. Finally, we show that the mixing time of the all-logit dynamics has the same twofold behavior that has been highlighted in the case of the one-logit: for some games it exponentially depends on the rationality level β, whereas for other games it can be upper bounded by a function independent from β.

MSC:

91A43	Games involving graphs
91A10	Noncooperative games
91A22	Evolutionary games
91A26	Rationality and learning in game theory
91D30	Social networks; opinion dynamics

Keywords:

logit dynamics; Markov chain; game dynamics; concurrent updates; noisy dynamics; reversibility

Full Text: DOI

References:

[1] Alós-Ferrer, C; Netzer, N, The logit-response dynamics, Games Econ. Behav., 68, 413-427, (2010) · Zbl 1207.91017

[2] Alós-Ferrer, C., Netzer, N.: Robust stochastic stability. ECON—working papers 063, Department of Economics, University of Zurich (Feb 2012)

[3] Anderson, SP; Goeree, JK; Holt, CA, Minimum-effort coordination games: stochastic potential and logit equilibrium, Games

Econ. Behav., 34, 177-199, (2001) · Zbl 1013.91006

[4] Anshelevich, E; Dasgupta, A; Tardos, É; Wexler, T, Near-optimal network design with selfish agents, Theory Comput., 4, 77-109, (2008) · Zbl 1213.68698

[5] Asadpour, A., Saberi, A.: On the inefficiency ratio of stable equilibria in congestion games. In: Proceedings of the 5th International Workshop on Internet and Network Economics (WINE'09), volume 5929 of Lecture Notes in Computer Science, pp 545-552. Springer (2009)

[6] Auletta, V., Ferraioli, D., Pasquale, F., Penna, P., Persiano, G.: Convergence to equilibrium of logit dynamics for strategic games. In: Proceedings of the 23rd ACM Symposium on Parallelism in Algorithms and Architectures (SPAA'11), pp. 197-206. ACM (2011) · Zbl 1349.91041

[7] Auletta, V., Ferraioli, D., Pasquale, F., Persiano, G.: Metastability of logit dynamics for coordination games. In: Proceedings of the ACM-SIAM Symposium on Discrete Algorithms (SODA'12), pp. 1006-1024. SIAM (2012)

[8] Auletta, V; Ferraioli, D; Pasquale, F; Persiano, G, Mixing time and stationary expected social welfare of logit dynamics, Theory Comput. Syst., 53, 3-40, (2013) · Zbl 1291.91026

[9] Babichenko, Y., Tamuz, O.: Graphical potential games. arXiv preprint arXiv:1405.1481 (2014) · Zbl 1369.91033

[10] Bala, V; Goyal, S, A noncooperative model of network formation, Econometrica, 68, 1181-1229, (2000) · Zbl 1022.91047

[11] Baron, R; Durieu, J; Haller, H; Solal, P, A note on control costs and logit rules for strategic games, J. Evolut. Econ., 12, 563-575, (2002)

[12] Baron, R; Durieu, J; Haller, H; Solal, P, Control costs and potential functions for spatial games, Int. J. Game Theory, 31, 541-561, (2003) · Zbl 1052.91024

[13] Berger, N; Kenyon, C; Mossel, E; Peres, Y, Glauber dynamics on trees and hyperbolic graphs, Probab. Theory Relat. Fields, 131, 311-340, (2005) · Zbl 1075.60003

[14] Bindel, D., Kleinberg, J.M., Oren, S.: How bad is forming your own opinion? In: Proceedings of the 52nd IEEE Annual Symposium on Foundations of Computer Science (FOCS'11), pp. 57-66 (2011) · Zbl 1292.91148

[15] Blume, LE, The statistical mechanics of strategic interaction, Games Econ. Behav., 5, 387-424, (1993) · Zbl 0797.90123

[16] Borgs, C., Chayes, J.T., Ding, J., Lucier, B.: The hitchhiker's guide to affiliation networks: a game-theoretic approach. In: Proceedings of the 2nd Symposium on Innovation in Computer Science (ICS'11), pp. 389-400. Tsinghua University Press (2011) · Zbl 0962.91008

[17] Borgs, C., Chayes, J.T., Karrer, B., Meeder, B., Ravi, R., Reagans, R., Sayedi, A.: Game-theoretic models of information overload in social networks. In: Proceedings of the 7th Workshop on Algorithms and Models for the Web Graph (WAW'10), pp. 146-161 (2010) · Zbl 1310.91034

[18] Corbo, J., Parkes, D.C.: The price of selfish behavior in bilateral network formation. In: Proceedings of the 24th Annual ACM Symposium on Principles of Distributed Computing (PODC'05), pp. 99-107 (2005) · Zbl 1314.91051

[19] Cournot, A.A.: Recherches sur le Principes mathematiques de la Theorie des Richesses. L. Hachette, Paris (1838) · Zbl 0174.51801

[20] Ellison, G, Learning, local interaction, and coordination, Econometrica, 61, 1047-1071, (1993) · Zbl 0802.90143

[21] Fabrikant, A., Luthra, A., Maneva, E.N., Papadimitriou, C.H., Shenker, S.: On a network creation game. In: Proceedings of the 22nd Annual ACM Symposium on Principles of Distributed Computing (PODC'03), pp. 347-351 (2003) · Zbl 1322.91013

[22] Ferraioli, D., Goldberg, P., Ventre, C.: Decentralized dynamics for finite opinion games. In: Proceedings of the 5th Internationl Symposium on Algorithmic Game Theory (SAGT'12), pp. 144-155. Springer, Berlin (2012) · Zbl 1284.91019

[23] Fudenberg, D., Levine, D.K.: The Theory of Learning in Games. MIT Press, Cambridge (1998) · Zbl 0939.91004

[24] Fudenberg, D., Tirole, J.: Game Theory. MIT Press, Cambridge (1992) · Zbl 1339.91001

[25] Goeree, JK; Holt, CA, Stochastic game theory: for playing games, not just for doing theory, Proc. Natl. Acad. Sci., 96, 10564-10567, (1999) · Zbl 0962.91008

[26] John, C.: Harsanyi and Reinhard Selten. A General Theory of Equilibrium Selection in Games. MIT Press, Cambridge (1988)

[27] Hart, S; Mas-Colell, A, A general class of adaptive procedures, J. Econ. Theory, 98, 26-54, (2001) · Zbl 0994.91007

[28] Jackson, MO; Wolinsky, A, A strategic model of social and economic networks, J. Econ. Theory, 71, 44-74, (1996) · Zbl 0871.90144

[29] Kelly, F.: Reversibility and Stochastic Networks. Cambridge University Press, Cambridge (2011) · Zbl 1260.60001

[30] Kleinberg, J.M., Oren, S.: Mechanisms for (mis)allocating scientific credit. In: Proceedings of the 43rd ACM Symposium on Theory of Computing (STOC'11), pp. 529-538 (2011) · Zbl 1288.91027

[31] Landau, L.D., Lifshitz, E.M.: Statistical Physics, vol. 5. Elsevier Science, Burlington (1996) · Zbl 0080.19702

[32] Levin, D., Peres, Y., Wilmer, E.L.: Markov Chains and Mixing Times. American Mathematical Society, Providence (2008)

[33] Levin, DA; Luczak, M; Peres, Y, Glauber dynamics for the Mean-field Ising model: cut-off, critical power law, and metastability, Probab. Theory Relat. Fields, 146, 223-265, (2010) · Zbl 1187.82076

[34] Martinelli, F.: Lectures on Glauber dynamics for discrete spin models. In: Lectures on Probability Theory and Statistics, volume 1717 of Lecture Notes in Mathematics, pp. 93-191. Springer, Berlin Heidelberg (1999) · Zbl 1051.82514

[35] McFadden, D.L.: Conditional logit analysis of qualitative choice behavior. In: Frontiers in Econometrics, pp. 105-142. Academic Press, New York (1974) · Zbl 0960.91016

[36] Mitzenmacher, M., Upfal, E.: Probability and Computing: Randomized Algorithms and Probabilistic Analysis. Cambridge University Press, Cambridge (2005) · Zbl 1092.60001

[37] Monderer, D; Shapley, LS, Potential games, Games Econ. Behav., 14, 124-143, (1996) · Zbl 0862.90137

[38] Montanari, A., Saberi, A.: Convergence to equilibrium in local interaction games. In: Proceedings of 50th Annual IEEE Symposium on Foundations of Computer Science (FOCS'09), pp. 303-312 (2009) · Zbl 1292.91036

[39] Morris, S, Contagion, Rev. Econ. Stud., 67, 57-78, (2000) · Zbl 0960.91016

[40] Peyton, HP; Blume, LE (ed.); Durlauf, SN (ed.), The diffusion of innovations in social networks, No. III, (2003), Oxford

[41] Rosenthal, RW, A class of games possessing pure-strategy Nash equilibria, Int. J. Game Theory, 2, 65-67, (1973) · Zbl 0259.90059

[42] Sandholm, William H.: Population Games and Evolutionary Dynamics. MIT Press, Cambridge (2010) · Zbl 1208.91003

[43] Trevisan, L.: Max cut and the smallest eigenvalue. In: Proceedings of the 41st Annual ACM Symposium on Theory of Computing, (STOC'09), pp 263-272. ACM (2009) · Zbl 1304.05140

[44] Weidenholzer, S, Coordination games and local interactions: a survey of the game theoretic literature, Games, 1, 551-585, (2010) · Zbl 1311.91059

[45] Wolpert, D.H.: Information theory—the bridge connecting bounded rational game theory and statistical physics. In: Braha, D., Minai, A.A., Bar-Yam, Y. (eds.) Complex Engineered Systems. Understanding Complex Systems, vol. 14, pp. 262-290. Springer Berlin Heidelberg (2006) · Zbl 0871.90144