Let $C(K)$ denote the space of all continuous real-valued functions on a compact subset $K$ of $\mathbb{R}^s$ ($s \geq 1$), satisfying $K = \text{int}(K)$. Let $W_\infty = \{ w \in L_\infty(K) : w > 0 \text{ on } K \}$, where $\|f\|_w = \int_K |f|w \ d\mu$ ($\mu$ denotes the Lebesgue measure in $\mathbb{R}^s$). A finite dimensional subspace $U$ of $C(K)$ is said to be Chebyshev in $C_w(K)$ if every $f \in C(k)$ has a unique best approximation from $U$ with respect to the above norm. For $f \in C(K)$ let $Z(f) = \{ x \in K : f(x) = 0 \}$. For a subspace $U$ of $C(K)$ let $U^* = \{ g \in C(K) : |g| \equiv |u| \text{ on } K \text{ for some } u \in U \}$. A finite-dimensional subspace $U$ of $C(K)$ is called an $A$-space (or is said to have the $A$-property) if for any $g \in U^* \setminus \{0\}$ there exists $u \in U \setminus \{0\}$ such that $u = 0$ a.e. on $Z(g)$ and $ug \geq 0$ on $K$. Finally, let us denote by (S) the following condition on a subset $W$ of $W_\infty$: “For a bounded measurable function $q$, $\int_K wq \ d\mu \geq 0$ for all $w \in W$ implies $q \geq 0$ a.e. on $K$.” In this paper the author characterizes $W \subseteq W_\infty$ so that the $A$-property is necessary for $U$ to be Chebyshev in $C_w(K)$ for all $w \in W$. He proves that if $W$ is a convex cone in $W_\infty$ satisfying condition (S) then every finite-dimensional subspace $U$ of $C(K)$ that is Chebyshev in $C_w(K)$ for all $w \in W$ is an $A$-space. He also proves that a finite-dimensional subspace $U$ of $C(K)$ is Chebyshev in $C_w(K)$ for all $w \in W_\infty$ if and only if for every $g \in U^* \setminus \{0\}$, 0 is not a best approximation to $g$ from $U$ relative to norm $\| \cdot \|_w$.

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MSC:
41A52 Uniqueness of best approximation
41A65 Abstract approximation theory (approximation in normed linear spaces and other abstract spaces)
41A50 Best approximation, Chebyshev systems

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