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**Global regularity results for the 2D Boussinesq equations with partial dissipation.** (English)

Zbl 1328.35161

J. Differ. Equations 260, No. 2, 1893-1917 (2016).

Summary: The two-dimensional (2D) incompressible Boussinesq equations model geophysical fluids and play an important role in the study of the Raleigh-Bernard convection. Mathematically this 2D system retains some key features of the 3D Navier-Stokes and Euler equations such as the vortex stretching mechanism. The issue of whether the 2D Boussinesq equations always possess global (in time) classical solutions can be difficult when there is only partial dissipation or no dissipation at all. This paper obtains the global regularity for two partial dissipation cases and proves several global a priori bounds for two other prominent partial dissipation cases. These results take us one step closer to a complete resolution of the global regularity issue for all the partial dissipation cases involving the 2D Boussinesq equations.

**MSC:**

- 35Q35 PDEs in connection with fluid mechanics
- 35B65 Smoothness and regularity of solutions to PDEs
- 35Q85 PDEs in connection with astronomy and astrophysics
- 76W05 Magnetohydrodynamics and electrohydrodynamics
- 85A30 Hydrodynamic and hydromagnetic problems in astronomy and astrophysics
- 76R10 Free convection
- 35B45 A priori estimates in context of PDEs

Cited in **34** Documents

**Keywords:**

Boussinesq equations; global regularity; partial dissipation

**Full Text:** [DOI](#)

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