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Greatest fixed points of probabilistic min/max polynomial equations, and reachability for branching Markov decision processes. (English) Zbl 1394.60084

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Summary: We give polynomial time algorithms for quantitative (and qualitative) reachability analysis for branching Markov decision processes (BMDPs). Specifically, given a BMDP, and given an initial population, where the objective of the controller is to maximize (or minimize) the probability of eventually reaching a population that contains an object of a desired (or undesired) type, we give algorithms for approximating the supremum (infimum) reachability probability, within desired precision $\epsilon > 0$, in time polynomial in the encoding size of the BMDP and in $\log(1/\epsilon)$. We furthermore give P-time algorithms for computing ϵ-optimal strategies for both maximization and minimization of reachability probabilities. We also give P-time algorithms for all associated qualitative analysis problems, namely: deciding whether the optimal (supremum or infimum) reachability probabilities are 0 or 1. Prior to this paper, approximation of optimal reachability probabilities for BMDPs was not even known to be decidable.

Our algorithms exploit the following basic fact: we show that for any BMDP, its maximum (minimum) non-reachability probabilities are given by the greatest fixed point (GFP) solution $g^* \in [0,1]^n$ of a corresponding monotone max (min) probabilistic polynomial system of equations (max/min-PPS), $x = P(x)$, which are the Bellman optimality equations for a BMDP with non-reachability objectives. We show how to compute the GFP of max/min PPSs to desired precision in P-time.

For the entire collection see [Zbl 1316.68013].

MSC:

60J80 Branching processes (Galton-Watson, birth-and-death, etc.)

Cited in **4** Documents

Keywords:

branching Markov decision processes; probabilistic polynomial system; Bellman optimality

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