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On the general solution of the problem of the motion of a heavy rigid body in the Hess case. (English. Russian original) [Zbl 1397.70006](#)

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In this paper, the motion of a heavy rigid body in the Hess case is considered. The solutions of the Euler-Poisson equations as analytic functions of time with given asymptotic behaviour in neighbourhoods of given singular points are presented. A representation of solutions of the Hess problem by means of the Riccati equation is used, which enables us to obtain a precise estimate for the boundary of the limiting periodic solutions.

Reviewer: [Clementina Mladenova \(Sofia\)](#)

MSC:

[70E15](#) Free motion of a rigid body
[70E20](#) Perturbation methods for rigid body dynamics
[70E40](#) Integrable cases of motion in rigid body dynamics
[70E50](#) Stability problems in rigid body dynamics

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[first integral](#); [Hess case of the Euler-Poisson equations](#); [asymptotic behaviour of solutions](#); [singular points of solutions](#); [analytic functions](#)

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