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Periodic solutions to operational differential equations with finite delay and impulsive conditions. (English) Zbl 1330.34119

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Summary: We study the following semilinear operational differential equation with finite delay and impulsive condition

$$u'(t) + Au(t) = f(t, u(t), u_t),\ t > 0, t \neq t_i,$$
$$u(s) = \phi(s),\ s \in [-r, 0],$$
$$\Delta u(t_i) = I_i(u(t_i)),\ i = 1, 2, \ldots, 0 < t_1 < t_2 < \cdots < \infty,$$

in a Banach space $(X, \|\cdot\|)$ with an unbounded operator A, where $r > 0$ is a constant and $u_t(s) = u(t+s)$, $s \in [-r, 0]$, which constitutes a finite delay, and $\Delta u(t_i) = u(t_i^+) - u(t_i^-)$ constitutes an impulsive condition which can be used to model more physical phenomena than the traditional initial value problems. We assume that $f(t, u, w)$ is T-periodic in t and then prove under some conditions that if solutions of the equation are ultimate bounded, then the operational differential equation has a T-periodic solution. The new result obtained here improves the corresponding result of [*J. Liang* et al., Nonlinear Anal., Theory Methods Appl., Ser. A, Theory Methods 74, No. 17, 6835–6842 (2011; Zbl 1242.34134)] by eliminating an assumption. Moreover, our arguments of proving the result are suitable for many other problems associated with impulsive conditions.

MSC:

34K45 Functional-differential equations with impulses
34G20 Nonlinear differential equations in abstract spaces

Keywords:

periodic solutions; impulsive conditions; delay

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