Shutov, A. V.

Trigonometric sums over one-dimensional quasilattices of arbitrary codimension. (English. Russian original) Zbl 1331.11062

Math. Notes 97, No. 5, 791-802 (2015); translation from Mat. Zametki 97, No. 5, 781-793 (2015).

The author discuss special trigonometric sums of the form

$$f_n(\lambda) = \sum_{j=1}^{n} e(x_j\lambda).$$

Here e^x means $e^{2\pi ix}$ and x_j is a point of the set of points $\{x_n\}_{-\infty}^{\infty}$ connected with the components T_j of the d-dimensional torus $\mathbb{T}^d$, $\mathbb{T}^d = T_0 \sqcup T_1 \cdots \sqcup T_d$, discussed by *V. G. Zhuravlev* [St. Petersbg. Math. J. 24, No. 1, 71–97 (2013; Zbl 1273.11121); translation from Algebra Anal. 24, No. 1, 95–130 (2012)].

The author defines the quasilattice $Q = Q(\alpha, l_0, \dots, l_d)$, where $l_0, l_1, \dots, l_d$ are pairwise different positive numbers and considers the same set of points $\{x_n\}_{-\infty}^{\infty}$, given by the conditions $x_{-1} = 0$, $x_{n+1} = x_n + l_j$ and $\alpha = (\alpha_1, \dots, \alpha_d)$ is a vector in $\mathbb{R}^d$.

The author proves the following theorem: Let $h_Q = \sum_{j=0}^{d} l_j \frac{\mathrm{vol}(T_j)}{\mathrm{vol}(T)}$.

1) If $h_Q\lambda \notin \sum_{j=1}^{d} \alpha_j\mathbb{Q} + \mathbb{Q}$, then $f_n(\lambda) = o(n)$.
2) If $h_Q\lambda = \frac{a_0 + \sum_{j=1}^{d} a_j\alpha_j}{b}$, $a_0, a_1, \dots, a_d, b \in \mathbb{Z}$, $(a_1, \dots, a_d, b) = 1$, $|b| > 1$, then $f_n(\lambda) = o(n)$.
3) If $h_Q\alpha = a_0 + \sum_{j=1}^{d} a_j\alpha_j$ and $a_0, a_1, \dots, a_d \in \mathbb{Z}$, then $f_n(\lambda) = c_{Q,\lambda}n + O(\Delta_2(\tilde{\alpha}, n))$, where $c_{Q,\lambda}$ is an effectively computable constant and $\Delta_2(\tilde{\alpha}, n)$ is explicitly determined in the text.

Reviewer: Tonko Tonkov (Sofia)

MSC:

11L03 Trigonometric and exponential sums, general

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trigonometric sum; quasilattice; codimension; bounded remainder set; tiling of the torus; Weyl's uniform distribution theorem; averaged lattice value; Koksma-Hlawka inequality; orbit structure

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