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Solution method for the transformed time-dependent Michaelis-Menten enzymatic reaction model. (English) Zbl 1331.92061

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Summary: The dynamic form of the Michaelis-Menten enzymatic reaction equations provides a time-dependent model in which a substrate S reacts with an enzyme E to form a complex C which is in turn converted into a product P and the enzyme E. In the recent paper [the second and the third author, J. Math. Chem. 52, No. 1, 222–230 (2014; Zbl 1311.92072)], it was shown that this system of four nonlinear equations can be reduced to a single nonlinear differential equation, which is simpler to solve numerically than the system of four equations. Qualitative properties of solutions were discussed, and stability results were given. In the present paper, we apply the optimal homotopy analysis method to the solution of this problem in order to obtain quantitative results. To do so, we transform the governing equation into a form that is more amenable to analysis. From the homotopy solutions, we are then able to study the effects of the model parameters on the solutions to the dynamic Michaelis-Menten enzymatic reaction equations. The results demonstrate the accuracy and efficiency of the approach, with residual errors of 10^{-6}–10^{-10} by considering relatively few iterations of the method. Therefore, the optimal homotopy analysis method is shown to be a rather useful tool for constructing analytical solutions to the dynamic Michaelis-Menten enzymatic reaction equations.

MSC:

92C45 Kinetics in biochemical problems (pharmacokinetics, enzyme kinetics, etc.)

Keywords:

dynamic Michaelis-Menten model; nonlinear dynamics; stability; enzyme reactions

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