Bestvina, Mladen; Reynolds, Patrick
The boundary of the complex of free factors. (English) Zbl 1337.20040
Duke Math. J. 164, No. 11, 2213-2251 (2015).

Let F_N be the free group of free rank N. With F_N is associated a complex $\mathcal{F} = \mathcal{F}_N$ of free factors. Vertices of $\mathcal{F}_N$ are conjugacy classes of non trivial proper free factors of F_N and higher-dimensional simplices correspond to chains of inclusions of free factors. $\mathcal{F}_N$ is equipped with the simplicial metric and in [*M. Bestvina* and *M. Feighn*, Adv. Math. 256, 104-155 (2014; Zbl 1348.20028); corrigendum 259, 843 (2014)] it is proved that $\mathcal{F}_N$ is Gromov hyperbolic. In the present paper the authors give a concrete description of the boundary $\partial\mathcal{F}_N$ of $\mathcal{F}_N$.

Before stating the main result we quote the necessary definitions and terminology. The unprojectivized outer space of rank N, denoted cv_N, is the topological space whose underlying set consists of free, minimal, discrete, isometric actions of F_N on $\mathbb{R}$-trees. A minimal F_N-tree is completely determined by its translation length function [*M. Culler* and *J. W. Morgan*, Proc. Lond. Math. Soc., III. Ser. 55, 571-604 (1987; Zbl 0658.20021)]. This gives an inclusion $cv_N \subseteq \mathbb{R}^{F_N}$ and a topology on cv_N. Let $\partial cv_N = \overline{cv_N} \setminus cv_N$ denote the boundary of cv_N, where $\overline{cv_N}$ is the closure of cv_N in $\mathbb{R}^{F_N}$.

The image of cv_N in the projective space $\mathbb{PR}^{F_N}$ is the Culler-Vogtmann Outer space CV_N and the boundary $\partial CV_N = \overline{CV_N} \setminus CV_N$ is the image of $\partial cv_N = \overline{cv_N} \setminus cv_N$.

Let ∂F_N be the Gromov boundary of F_N, that is the boundary of any Cayley graph of F_N. Let $\partial^2(F_N) = \partial F_N \times \partial F_N \setminus \Delta$, where Δ is the diagonal. The left action of F_N on a Cayley graph induces actions by homeomorphisms of F_N on ∂F_N and on $\partial^2(F_N)$. Let $i\colon \partial^2(F_N) \to \partial^2(F_N)$ denote the involution (the flip) that exchanges the factors. A lamination is a nonempty, closed F_N-invariant, i-invariant subset $L \subseteq \partial^2(F_N)$. The elements of a lamination X are called leaves and it is called arational if no leaf is carried by a proper free factor of F_N.

Associated to a $T \in \partial cv_N$ is a lamination $L(T)$, which is constructed as follows. Let $L_\varepsilon(T) = \overline{\{(g^{-\infty}, g^{\infty} \mid \ell_T(g) < \varepsilon\}}$, and define $L(T) = \bigcap_{\varepsilon>0} L_\varepsilon(T)$. Here, for a $1 \neq g \in F_N$, $g^{-\infty}$ and g^{∞} denote the attracting and repelling fixed points of g in ∂F_N and ℓ_T is a length function on T.

A tree $T \in \partial CV_N$ is called arational if the lamination $L(T)$ is arational. Let $\mathcal{AT} \subseteq \partial CV_N$ denote the set of arational trees, equiped with the subspace topology. Define a relation $\sim$ on $\mathcal{AT}$ by $S \sim T$ if and only if $L(S) = L(T)$, and give $\mathcal{AT}/\sim$ the quotient topology.

Now we can state the main theorem. Theorem. The space $\partial\mathcal{F}$ is homeomorphic to $\mathcal{AT}/\sim$.

The arguments of the authors use the geometry of Outer space and folding paths as developed by *M. Bestvina* and *M. Feighn* [op. cit.] and the structure theory of trees in ∂CV_N developed by Coulbois, Hilion, Lustig, and Reynolds in recent series of papers.

As the authors point out the theorem above is a very strong analogy of *E. Klarreich*'s description, [in The boundary at infinity of the complex of curves and the relative Teichmüller space, preprint `https://pressfolios-production.s3.amazonaws.com/uploads/story/story-pdf/145710/1457101434403642.pdf`], of the boundary $\partial\mathcal{C}(S)$ of the complex of curves $\mathcal{C}(S)$ associated to a nonexceptional surface S.

Reviewer: Dimitrios Varsos (Athína)

MSC:

20F65 Geometric group theory
20E05 Free nonabelian groups
20E08 Groups acting on trees
20F67 Hyperbolic groups and nonpositively curved groups
37A25 Ergodicity, mixing, rates of mixing
37B10 Symbolic dynamics
57M07 Topological methods in group theory

Keywords:

free groups; complexes of free factors; Gromov hyperbolic complexes; boundaries; outer space; translation length functions; Cayley graphs; arational trees; laminations; isometric actions on trees

Full Text: DOI arXiv