Khodaei, Hamid

On the stability of additive, quadratic, cubic and quartic set-valued functional equations. (English) Zbl 1330.39029

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Let X be a real vector space and Y a Banach space. By $C_{cb}(Y)$ we denote the set of all nonempty, closed, bounded and convex subsets of Y. By $A \oplus B$ we mean the closure of $A+B$. For $f : X \to C_{cb}(Y)$, a fixed integer $a > 1$ and $m = 1, 2, 3, 4$, the following equation is considered and its stability is proved.

$$f(ax+y) \oplus f(ax-y) = a^{m-2}[f(x+y) \oplus f(x-y)] \oplus 2(a^2-1)\left[a^{m-2}f(x) \oplus \frac{(m-2)(1-(m-2)^2)}{6}f(y)\right]$$

for all $x, y \in X$. Namely, if the Hausdorff distance between the left and right hand sides of the above equation is bounded by a suitably contractively subhomogeneous function, then there exists a unique additive, quadratic, cubic or quartic (for $m = 1, 2, 3, 4$, respectively) mapping which is sufficiently close to f.

Reviewer: Jacek Chmieliński (Kraków)

MSC:

39B82 Stability, separation, extension, and related topics for functional equations

39B52 Functional equations for functions with more general domains and/or ranges

54C60 Set-valued maps in general topology

39B55 Orthogonal additivity and other conditional functional equations

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set-valued mapping; subhomogeneous mapping; expansively superhomogeneous mapping; stability of functional equations; fixed point method; additive, quadratic, cubic and quartic set-valued functional equations; Banach space

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