Béré, Côme J. A.; Ouedraogo, M. Françoise; Pilabré, Nakelgbamba B.
On the existence of ad-nilpotent elements. (English) Zbl 1350.17004
Afr. Mat. 26, No. 5-6, 813-823 (2015).

For an arbitrary element x of a Leibniz algebra L consider two operators:

$$\mathrm{ad}_x : L \to L, \quad y \to [y, x]$$

and

$$\mathrm{Ad}_x : L \to L, \quad y \to [x, y].$$

Clearly, ad_x and Ad_x are derivation and anti-derivation of L, respectively.

For these operators the following relations hold true:

$$\begin{aligned} \mathrm{ad}_{[x,y]} &= \mathrm{ad}_y \mathrm{ad}_x - \mathrm{ad}_x \mathrm{ad}_y, \\ \mathrm{Ad}_{[x,y]} &= \mathrm{ad}_y \mathrm{Ad}_x - \mathrm{Ad}_x \mathrm{ad}_y, \\ \mathrm{Ad}_{[x,y]} &= \mathrm{ad}_y \mathrm{Ad}_x + \mathrm{Ad}_x \mathrm{Ad}_y, \\ 0 &= \mathrm{Ad}_x \mathrm{Ad}_y + \mathrm{Ad}_x \mathrm{ad}_y. \end{aligned}$$

A (bi)module M over a Leibniz algebra L is a vector space with two (left l and right r) actions, satisfying the above relations.

Let L be a Leibniz algebra and M be L-(bi)module. We denote by $\mathrm{Ess}(M)$ the subspace of M spanned by elements of the type $l_x(v) + r_x(v) = xv + vx$ for all $(x, v) \in L \times M$.

In the present paper the authors prove the invariance of $\mathrm{Ess}(L)$ under derivations of L and $\mathrm{Ess}(L) \subseteq \mathrm{Ker}\widetilde{D}$ for any anti-derivation $\widetilde{D}$ of L. Moreover, the embedding $[L_\lambda, L_\mu] \subseteq L_{\lambda+\mu} + \mathrm{Ess}(L)$, where L_λ, L_μ are weight spaces with respect to a given anti-derivation, is established.

The main results of the paper is the following:

Theorem. Let L be a Leibniz algebra over an algebraically closed field. Let X be a non-empty subset of L such that for every $x \in X$, all eigenvectors of ad_x (correspondingly, of Ad_x) lie in X. Then ad_y (correspondingly, Ad_x) is nilpotent for some $y \in X$.

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MSC:

17A32 Leibniz algebras
17B30 Solvable, nilpotent (super)algebras

Keywords:

Lie algebras; Leibniz algebras; ad-nilpotent; derivation; anti-derivation; module

Full Text: DOI

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