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**Stabilized schemes for the hydrostatic Stokes equations.** (English) Zbl 1328.35168  
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This article deals with some numerical schemes for the finite element approximation of the hydrostatic Stokes equations that are used in oceanography. Considering appropriate functional spaces for the velocity and the pressure, the problem is reformulated by adding a residual term to the vertical momentum equation, a term that stabilizes the vertical velocity. This technique allows to recover partially the  $H^1$ -coercivity, which disappears in hydrostatic problems, to prove stability for Stokes-stable finite element combinations and to provide some error estimates.

In the next section another residual term is added to the continuity equation in order to obtain a better approximation of the vertical derivative of the pressure. The well-posedness of the discrete scheme is proved and some error estimates are established, including the  $L^2$ -norm of the vertical derivative of the pressure.

Finally, some numerical simulations are presented, in order to illustrate the advantages of the schemes previously described.

Reviewer: [Ruxandra Stavre \(București\)](#)

**MSC:**

- [35Q35](#) PDEs in connection with fluid mechanics
- [65N12](#) Stability and convergence of numerical methods for boundary value problems involving PDEs
- [65N15](#) Error bounds for boundary value problems involving PDEs
- [65N30](#) Finite element, Rayleigh-Ritz and Galerkin methods for boundary value problems involving PDEs
- [86A05](#) Hydrology, hydrography, oceanography
- [76D07](#) Stokes and related (Oseen, etc.) flows

Cited in 4 Documents

**Keywords:**

[hydrostatic Stokes equations](#); [finite-elements](#); [stabilized schemes](#)

**Software:**

[FreeFem++](#)

**Full Text:** [DOI](#)

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