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Remarks on global regularity of the 2D Boussinesq equations with fractional dissipation.
(English) [Zbl 1405.35171](#)

Nonlinear Anal., Theory Methods Appl., Ser. A, Theory Methods 125, 715-724 (2015).

Summary: In this paper, we are interested in the study of the Cauchy problem to the two-dimensional (2D) incompressible Boussinesq equations with fractional dissipation. By making use of the nonlinear lower bounds for the fractional Laplacian established in [*P. Constantin* and *V. Vicol*, *Geom. Funct. Anal.* 22, No. 5, 1289–1321 (2012; [Zbl 1256.35078](#))], we establish the global regularity of the smooth solutions of the 2D Boussinesq equations with a new range of fractional powers of the Laplacian. This result significantly improves the recent works of Constantin and Vicol [*loc. cit.*] and *W. Yang et al.* [*J. Differ. Equations* 257, No. 11, 4188–4213 (2014; [Zbl 1300.35108](#))].

MSC:

[35Q35](#) PDEs in connection with fluid mechanics

[35B65](#) Smoothness and regularity of solutions to PDEs

[76D03](#) Existence, uniqueness, and regularity theory for incompressible viscous fluids

Cited in **9** Documents

Keywords:

2D Boussinesq equations; fractional dissipation; global regularity

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References:

- [1] D. Adhikari, C. Cao, H. Shang, J. Wu, X. Xu, Z. Ye, Global regularity results for the 2D Boussinesq equations with partial dissipation (2014) submitted for publication. · [Zbl 1328.35161](#)
- [2] Adhikari, D.; Cao, C.; Wu, J.; Xu, X., Small global solutions to the damped two-dimensional Boussinesq equations, *J. Differential Equations*, 256, 3594-3613, (2014) · [Zbl 1290.35193](#)
- [3] Brézis, H.; Gallouet, T., Nonlinear Schrödinger evolution equations, *Nonlinear Anal.*, 4, 4, 677-681, (1980) · [Zbl 0451.35023](#)
- [4] Cannon, J.; DiBenedetto, E., The initial value problem for the Boussinesq equation with data in L^p , (*Lecture Notes in Mathematics*, vol. 771, (1980), Springer Berlin), 129-144
- [5] Cao, C.; Wu, J., Global regularity for the 2D anisotropic Boussinesq equations with vertical dissipation, *Arch. Ration. Mech. Anal.*, 208, 985-1004, (2013) · [Zbl 1284.35140](#)
- [6] Chae, D., Global regularity for the 2D Boussinesq equations with partial viscosity terms, *Adv. Math.*, 203, 497-513, (2006) · [Zbl 1100.35084](#)
- [7] Chae, D.; Wu, J., The 2D Boussinesq equations with logarithmically supercritical velocities, *Adv. Math.*, 230, 1618-1645, (2012) · [Zbl 1248.35156](#)
- [8] Constantin, P.; Vicol, V., Nonlinear maximum principles for dissipative linear nonlocal operators and applications, *Geom. Funct. Anal.*, 22, 1289-1321, (2012) · [Zbl 1256.35078](#)
- [9] Córdoba, A.; Córdoba, D., A maximum principle applied to quasi-geostrophic equations, *Comm. Math. Phys.*, 249, 3, 511-528, (2004) · [Zbl 1309.76026](#)
- [10] Danchin, R., Remarks on the lifespan of the solutions to some models of incompressible fluid mechanics, *Proc. Amer. Math. Soc.*, 141, 1979-1993, (2013) · [Zbl 1283.35080](#)
- [11] Danchin, R.; Paicu, M., Global well-posedness issues for the inviscid Boussinesq system with Yudovich's type data, *Comm. Math. Phys.*, 290, 1-14, (2009) · [Zbl 1186.35157](#)
- [12] Danchin, R.; Paicu, M., Global existence results for the anisotropic Boussinesq system in dimension two, *Math. Models Methods Appl. Sci.*, 21, 421-457, (2011) · [Zbl 1223.35249](#)
- [13] KC, D.; Regmi, D.; Tao, L.; Wu, J., The 2D Euler-Boussinesq equations with a singular velocity, *J. Differential Equations*, 257, 82-108, (2014) · [Zbl 1291.35221](#)
- [14] Hajaiej, H.; Molinet, L.; Ozawa, T.; Wang, B., Sufficient and necessary conditions for the fractional Gagliardo-Nirenberg inequalities and applications to Navier-Stokes and generalized boson equations, (*Ozawa, T.; Sugimoto, M., RIMS Kkyroku Bessatsu B26: Harmonic Analysis and Nonlinear Partial Differential Equations*, vol. 5, (2011)), 159-175 · [Zbl 1270.42026](#)

- [15] Hmidi, T., On a maximum principle and its application to the logarithmically critical Boussinesq system, *Anal. PDE*, 4, 247-284, (2011) · [Zbl 1264.35173](#)
- [16] Hmidi, T.; Keraani, S.; Rousset, F., Global well-posedness for a Boussinesq-Navier-Stokes system with critical dissipation, *J. Differential Equations*, 249, 2147-2174, (2010) · [Zbl 1200.35228](#)
- [17] Hmidi, T.; Keraani, S.; Rousset, F., Global well-posedness for Euler-Boussinesq system with critical dissipation, *Comm. Partial Differential Equations*, 36, 420-445, (2011) · [Zbl 1284.76089](#)
- [18] Hou, T. Y.; Li, C., Global well-posedness of the viscous Boussinesq equations, *Discrete Contin. Dyn. Syst.*, 12, 1-12, (2005) · [Zbl 1274.76185](#)
- [19] Jiu, Q.; Miao, C.; Wu, J.; Zhang, Z., The 2D incompressible Boussinesq equations with general critical dissipation, *SIAM J. Math. Anal.*, 46, 3426-3454, (2014) · [Zbl 1319.35193](#)
- [20] Jiu, Q.; Wu, J.; Yang, W., Eventual regularity of the two-dimensional Boussinesq equations with supercritical dissipation, *J. Nonlinear Sci.*, 25, 37-58, (2015) · [Zbl 1311.35221](#)
- [21] Lai, M.; Pan, R.; Zhao, K., Initial boundary value problem for two-dimensional viscous Boussinesq equations, *Arch. Ration. Mech. Anal.*, 199, 739-760, (2011) · [Zbl 1231.35171](#)
- [22] Larios, A.; Lunasin, E.; Titi, E. S., Global well-posedness for the 2D Boussinesq system with anisotropic viscosity and without heat diffusion, *J. Differential Equations*, 255, 2636-2654, (2013) · [Zbl 1284.35343](#)
- [23] Li, J.; Titi, E., Global well-posedness of the 2D Boussinesq equations with vertical dissipation · [Zbl 1336.35297](#)
- [24] Liu, X.; Wang, M.; Zhang, Z., Local well-posedness and blowup criterion of the Boussinesq equations in critical Besov spaces, *J. Math. Fluid Mech.*, 12, 280-292, (2010) · [Zbl 1195.76136](#)
- [25] Majda, A.; Bertozzi, A., *Vorticity and incompressible flow*, (2001), Cambridge University Press Cambridge
- [26] Miao, C.; Xue, L., On the global well-posedness of a class of Boussinesq-Navier-Stokes systems, *NoDEA Nonlinear Differential Equations Appl.*, 18, 707-735, (2011) · [Zbl 1235.76020](#)
- [27] Pedlosky, J., *Geophysical fluid dynamics*, (1987), Springer-Verlag New York · [Zbl 0713.76005](#)
- [28] Stefanov, A.; Wu, J., A global regularity result for the 2D Boussinesq equations with critical dissipation
- [29] Wu, J.; Xu, X., Well-posedness and inviscid limits of the Boussinesq equations with fractional Laplacian dissipation, *Nonlinearity*, 2215-2232, (2014) · [Zbl 1301.35115](#)
- [30] Wu, J.; Xu, X.; Ye, Z., Global smooth solutions to the n-dimensional damped models of incompressible fluid mechanics with small initial datum, *J. Nonlinear Sci.*, 25, 157-192, (2015) · [Zbl 1311.35236](#)
- [31] Wu, G.; Xue, L., Global well-posedness for the 2D inviscid Bénard system with fractional diffusivity and Yudovich's type data, *J. Differential Equations*, 253, 100-125, (2012) · [Zbl 1305.35119](#)
- [32] Xu, X., Global regularity of solutions of 2D Boussinesq equations with fractional diffusion, *Nonlinear Anal.*, 72, 677-681, (2010) · [Zbl 1177.76024](#)
- [33] Xu, X.; Xue, L., Yudovich type solution for the 2D inviscid Boussinesq system with critical and supercritical dissipation, *J. Differential Equations*, 256, 3179-3207, (2014) · [Zbl 1452.76030](#)
- [34] Xu, X.; Ye, Z., The lifespan of solutions to the inviscid 3D Boussinesq system, *Appl. Math. Lett.*, 26, 854-859, (2013) · [Zbl 1314.35113](#)
- [35] Yang, W.; Jiu, Q.; Wu, J., Global well-posedness for a class of 2D Boussinesq systems with fractional dissipation, *J. Differential Equations*, 257, 4188-4213, (2014) · [Zbl 1300.35108](#)
- [36] Ye, Z., Blow-up criterion of smooth solutions for the Boussinesq equations, *Nonlinear Anal.*, 110, 97-103, (2014) · [Zbl 1300.35109](#)
- [37] Z. Ye, X. Xu, L. Xue, On the global regularity of the 2D Boussinesq equations with fractional dissipation (2014) submitted for publication.

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